Mathcad sheet to calculate the compensation of the two type-I crystals for entanglement.
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$nm := 10^{-9} \cdot m$ $\quad \mu m := 10^{-6} \cdot m$ $\quad fs := 10^{-15} \cdot s$

Wavelength of the pump $\quad \lambda_p := 405 \cdot nm$ $\quad \lambda_{p2} := 408 \cdot nm$

Wavelength of the degenerate down-converted photons $\quad \lambda_{dc} := 2 \cdot \lambda_p$ $\quad \lambda_{dc} = 810 \cdot nm$

$\lambda_{dc2} := 2 \cdot \lambda_{p2}$ $\quad \lambda_{dc2} = 816 \cdot nm$

Index of refraction of BBO

$A_o := 2.7359$ $\quad B_o := 0.01878$ $\quad C_o := -0.01822$ $\quad D_o := -0.01354$ $\quad$ Kato IEE

$A_e := 2.3753$ $\quad B_e := 0.01224$ $\quad C_e := -0.01667$ $\quad D_e := -0.01516$

$A_o := 2.7405$ $\quad B_o := 0.0184$ $\quad C_o := -0.0179$ $\quad D_o := -0.0155$ $\quad$ Eimerl JAP

$A_e := 2.373$ $\quad B_e := 0.0128$ $\quad C_e := -0.0156$ $\quad D_e := -0.004$

Note : Different people use different constants. The differences are small but visible.

Ordinary index of refraction:

$$n_o(\lambda) := \left[A_o + \frac{B_o}{\left(\frac{\lambda}{\mu m}\right)^2 + C_o} + D_o \cdot \left(\frac{\lambda}{\mu m}\right)^2 \right]^{\frac{1}{2}}$$

Extraordinary index of refraction:

$$n_e(\lambda) := \left[A_e + \frac{B_e}{\left(\frac{\lambda}{\mu m}\right)^2 + C_e} + D_e \cdot \left(\frac{\lambda}{\mu m}\right)^2 \right]^{\frac{1}{2}}$$

Extraordinary index of refraction when the electric field is not strictly parallel to the optic axis:

$$n_{pe}(\theta_{pm}, \lambda_p) := \left(\frac{\cos(\theta_{pm})^2}{n_o(\lambda_p)^2} + \frac{\sin(\theta_{pm})^2}{n_e(\lambda_p)^2} \right)^{\frac{-1}{2}}$$

The phase-matching angle θ_pm is the one formed by the wave-vector and the optic axis.

$n_o(\lambda_{dc}) = 1.661$ index of DC photons $\quad n_e(\lambda_p) = 1.568$ index of pump with no phase matching

$n_{pe}(29 \cdot deg, \lambda_p) = 1.66$ index of pump with phase matching (phase-matching angle already known)

$$\lambda_i(\lambda_s) := \left(\frac{1}{\lambda_p} - \frac{1}{\lambda_s} \right)^{-1}$$

Wavelength (in vacuum) of the idler DC photon when the wavelength of the signal photon has been defined.

Wavevectors allowing for the doen-converted photons to be non-degenerate

$$k_p(\theta_{pm}) := \frac{2\cdot\pi\cdot n_{pe}(\theta_{pm},\lambda_p)}{\lambda_p} \qquad k_s(\lambda_s) := \frac{2\cdot\pi\cdot n_o(\lambda_s)}{\lambda_s} \qquad k_i(\lambda_s) := \frac{2\cdot\pi\cdot n_o(\lambda_i(\lambda_s))}{\lambda_i(\lambda_s)}$$

$$\theta_i(\theta_{pm},\lambda_s) := \operatorname{acos}\left(\frac{k_p(\theta_{pm})^2 - k_s(\lambda_s)^2 + k_i(\lambda_s)^2}{2\cdot k_p(\theta_{pm})\cdot k_i(\lambda_s)}\right)$$

Angle **inside the crystal** that the idler wave-vector forms with the pump, for given phase-matching angle
and wavelength of the idler.

$$\theta_i(29.1\cdot deg,\lambda_{dc}) = 1.812\cdot deg$$

$$\theta_i(29.1\cdot deg, 820\cdot nm) = 1.793\cdot deg$$

$$\theta'_i(\theta_{pm},\lambda_s) := \operatorname{asin}(n_o(\lambda_i(\lambda_s))\cdot\sin(\theta_i(\theta_{pm},\lambda_s)))$$

Angle that the idler forms outside the crystal.

$$\theta'_i(29.1\cdot deg,\lambda_{dc}) = 3.01\cdot deg$$

Small calculation: if the collimator has an acceptance angle of 2 mm (at 1 m from crystal), what wavelength range does it accept?

$$\Delta\theta_{coll} := \frac{2}{1000} \qquad \Delta\theta_{coll} = 0.115\cdot deg$$

$$\theta'_i(29.1\cdot deg, 906\cdot nm) - \theta'_i(29.1\cdot deg,\lambda_{dc}) = 0.112\cdot deg$$

Thus the collimation has an effective bandwidth of about 100 nm

Another small calculation: what extra crystal tilt will get our desired DC photons out of range of the collimator acceptance angle?

$$\theta'_i(29.14\cdot deg,\lambda_{dc}) - \theta'_i(29.1\cdot deg,\lambda_{dc}) = 0.139\cdot deg$$

A tenth of a degree makes a difference (0.04 deg in the calculation) --tuning the crystal is critical.

Now we solve for the phase-matching angle

For collinear down-conversion

$$\theta_{pm0} := \operatorname{root}(n_{pe}(\theta_{pmz},\lambda_p) - n_o(\lambda_{dc}),\theta_{pmz},0,90\cdot deg) \qquad \theta_{pm0} = 28.673\cdot deg$$

For down-conversion at +/- 3 degrees

$$\theta_{pm3} := \operatorname{root}\left(n_{pe}(\theta_{pmz},\lambda_p) - n_o(\lambda_{dc})\cdot\sqrt{1 - \frac{\sin(3\cdot deg)^2}{n_o(\lambda_{dc})^2}},\theta_{pmz},0,90\cdot deg\right) \qquad \theta_{pm3} = 29.097\cdot deg$$

For the second wavelength:

$$\theta_{pm32} := \text{root}\left(n_{pe}(\theta_{pmz}, \lambda_{p2}) - n_o(\lambda_{dc2})\cdot\sqrt{1 - \frac{\sin(3\cdot deg)^2}{n_o(\lambda_{dc2})^2}}, \theta_{pmz}, 0, 90\cdot deg\right) \qquad \theta_{pm32} = 28.893\cdot deg$$

$\theta_i(\theta_{pm3}, \lambda_{dc}) = 1.806\cdot deg$ double - check: DC angle inside the crystal

$\theta'_i(\theta_{pm3}, \lambda_{dc}) = 3\cdot deg$ double - check: DC angle outside the crystal

Now we need to compensate for the temporal mismatch when we use two type-! crystals for entanglement. Since we assume that the photons are wavepackets, we need to find group velocities.

$dl := 5\cdot nm$

$$dnedl(l) := \frac{-n_{pe}(\theta_{pm3}, l - dl) + n_{pe}(\theta_{pm3}, l + dl)}{2\cdot dl} \qquad dnodl(l) := \frac{-n_o(l - dl) + n_o(l + dl)}{2\cdot dl}$$

$dnedl(\lambda_{dc}) = -2.654 \times 10^4 \frac{1}{m}$ $dnodl(\lambda_{dc}) = -2.959 \times 10^4 \frac{1}{m}$

group velocity indices

$neg(\lambda) := n_{pe}(\theta_{pm3}, \lambda) - dnedl(\lambda)\cdot\lambda$ $nog(\lambda) := n_o(\lambda) - dnodl(\lambda)\cdot\lambda$

Group velocity Indices for different wavelengths

Wavelength	ne*-dne*dl l	no - dnodl l
$\lambda_p = 405\cdot nm$	$neg(\lambda_p) = 1.739$	$nog(\lambda_p) = 1.777$
$\lambda_{dc} = 810\cdot nm$	$neg(\lambda_{dc}) = 1.653$	$nog(\lambda_{dc}) = 1.685$
$\lambda_{p2} = 408\cdot nm$	$neg(\lambda_{p2}) = 1.737$	$nog(\lambda_{p2}) = 1.775$
$\lambda_{dc2} = 816\cdot nm$	$neg(\lambda_{dc2}) = 1.653$	$nog(\lambda_{dc2}) = 1.685$

$c = 2.998 \times 10^8 \frac{m}{s}$

$d := 0.5 \cdot mm$ Crystal thickness

time lag between the two DC pairs

$$\Delta t(\lambda_p) := \frac{d}{c} \cdot nog(\lambda_p) - \frac{d}{c} \cdot neg(2\lambda_p)$$

The delay is due to the mismatch of pump photons traveling as ordinary polarized in the first crystal and down-converted photons traveling as extraordinary. Strictly speaking we need to account the phase-matching angle, but will neglect that because it will be a small correction.

$\Delta t(\lambda_p) = 207.293 \cdot fs$ time delay between photon pairs $\Delta t(\lambda_{p2}) = 204.375 \cdot fs$ for λ_2

The coherence length is $$l_c(\lambda, \Delta\lambda) := \frac{\lambda^2}{\Delta\lambda}$$

so the wavepacket width is $$\Delta t_{wp}(\lambda, \Delta\lambda) := \frac{l_c(\lambda, \Delta\lambda)}{c}$$

$l_c(\lambda_{dc}, 40 \cdot nm) = 16.403 \cdot \mu m$ $l_c(\lambda_{dc}, 10 \cdot nm) = 65.61 \cdot \mu m$

Useful information for interferometers: the path length difference between the two arms has to be less than l_c

If the bandwidth of the pump photon is $\Delta\lambda_p := 0.5 \cdot nm$ then $\Delta t_{wp}(\lambda_p, \Delta\lambda_p) = 1.094 \times 10^3 \cdot fs$

or much worse if the bandwidth is larger $\Delta\lambda_p := 6 \cdot nm$ then $\Delta t_{wp}(\lambda_p, \Delta\lambda_p) = 91.188 \cdot fs$

This is much greater than the delay between the down-converted photons.

We need to precompensate for the temporal distinguishability with a quartz crystal

Interpolating data from www.sciner.com web page in quartz.org we get:

$$n_{oq}(\lambda) := 9.46503 \cdot 10^{-9} \cdot \left(\frac{\lambda}{nm}\right)^2 - 3.47588 \cdot 10^{-5} \cdot \frac{\lambda}{nm} + 1.56023 \qquad n_{oq}(\lambda_p) = 1.54770519$$

$$n_{eq}(\lambda) := 9.65822 \cdot 10^{-9} \cdot \left(\frac{\lambda}{nm}\right)^2 - 3.57864 \cdot 10^{-5} \cdot \frac{\lambda}{nm} + 1.56984 \qquad n_{eq}(\lambda_p) = 1.55693$$

$dl := 5 \cdot nm$

Group velocity indices

$$dneqdl(l) := \frac{n_{eq}(l + dl) - n_{eq}(l - dl)}{2 \cdot dl} \qquad dnoqdl(l) := \frac{n_{oq}(l + dl) - n_{oq}(l - dl)}{2 \cdot dl}$$

$$n_{eqg}(\lambda) := n_{eq}(\lambda) - dneqdl(\lambda) \cdot \lambda \qquad n_{oqg}(\lambda) := n_{oq}(\lambda) - dnoqdl(\lambda) \cdot \lambda$$

Compensating crystal thickness:

$$d_q(\lambda_p) := \frac{\Delta t(\lambda_p) \cdot c}{n_{eqg}(\lambda_p) - n_{oqg}(\lambda_p)}$$

$\Delta t(\lambda_p) = 207.293 \cdot fs$

I note that this is consistent with the results of Rangarajan, Goggin and Kwiat OE 17, 18920 (2009): for d=0.6mm they find 253 fs and we do too using Kato's indices, but find 249 with Eimerl.

$d_q(\lambda_p) = 6.488 \cdot mm$ $\qquad$ $d_q(\lambda_{p2}) = 6.397 \cdot mm$