

Resource Letter SPE-1: Single-Photon Experiments in the Undergraduate Laboratory

Enrique J. Galvez

Department of Physics and Astronomy, Colgate University, Hamilton, New York 13346

(Received 6 December 2013; accepted 9 April 2014)

This Resource Letter lists undergraduate-laboratory adaptations of landmark optical experiments on the fundamentals of quantum physics. Journal articles and websites give technical details of the adaptations, which offer students unique hands-on access to testing fundamental concepts and predictions of quantum mechanics. A selection of the original research articles that led to the implementations is included. These developments have motivated a rethinking of the way quantum mechanics is taught, so this Resource Letter also lists textbooks that provide these new approaches. © 2014 American Association of Physics Teachers.

[http://dx.doi.org/10.1119/1.4872135]

I. INTRODUCTION

Quantum mechanics is one of the most important physical theories of our times. With unparalleled accuracy and predictive power, it allows us to comprehend nature at its most fundamental level. At the same time, it forces us to sacrifice many of our deeply rooted beliefs. It maintains nonrealism: objects or systems do not have an inherent or defined reality; or indeterminism: once the initial conditions are set, the future properties and trajectories of quantum systems will be inherently unknowable. More strikingly, it predicts nonlocality: events in one location may instantly correlate with other events in a different location. At the heart of this conceptual challenge is quantum superposition and the interference of indistinguishable possibilities. Paraphrasing Niels Bohr: "Any one who is not shocked by quantum mechanics has not fully understood it."

Traditionally, one assimilates quantum concepts only after a thorough immersion in its mathematical architecture, with all of its elegance and abstraction. Strikingly, understanding the fundamental underpinnings is often avoided altogether, following the popular dictum: "Shut up and calculate." It is easy to become satisfied with calculating and not understanding, but this ignorance prevents the deeper understanding of the theory and its implications. Furthermore, mastering the fundamentals of quantum mechanics is essential in an era where technologies reach the quantum limit.

The traditional approach of learning quantum mechanics may be an unrealistic strategy for teaching undergraduates. How can we circumvent the mathematical formalism and still introduce the beautiful quirkiness of quantum mechanics in an intellectually honest and compelling way? How can we convince the student that those mathematical abstractions refer to reality?

Teaching materials and simulations can give the student a solid base for understanding quantum phenomena, but experimentation provides the decisive ingredient. Quantum mechanics predicts results that are hard to accept, and thus experiments and measurements provide the unquestionable evidence and the ultimate convincing power. The collection of works presented here concentrate on the conception, design, and execution of experiments that teach us about nature through the lens of quantum mechanics.

Experimental demonstrations of the fundamentals of quantum mechanics have a long history, but the pace has accelerated significantly in the last two decades, owing mostly to technological developments. Students in an afternoon can now recreate experiments that once were monumental achievements of research ingenuity and expertise, taking years of development. The importance of this lab experience is not what students do, but what they are prompted to ponder. We aim for students to say to one another: "Did we just measure that?" This is likely what those who originally performed the landmark experiments thought.

The technology uses light—a source of pairs of photons through the process known as spontaneous parametric downconversion. In this process, a single photon is converted into a pair of photons that are correlated and entangled in subtle ways. These correlations fueled the landmark experiments of the past three decades exploring the fundamentals of quantum physics. Along with the advances in the research came technological developments that made undergraduate implementations feasible: packaged single-photon detectors, computerized electronics fitting in a single electronic board, and short-wavelength diode lasers that are also sold as pointers. Implementations use standard optical components (mirrors, beam splitters, polarizers, and other similar components) to manipulate the photon correlations and test fundamental quantum principles. In all, they simplify the apparatus to a setup that fits in a 2-ft \times 5-ft optical breadboard. Detectors outputting a digital pulse per photon detection and digital electronics make data acquisition relatively straightforward. One intrinsic advantage of these types of experiments is that the data are collected one quantum at a time-the same way that we understand the mathematics. Data can then be logged and graphed directly with no need for approximations or sophisticated analysis to understand the results. Thus, these experiments offer exciting new possibilities to teach introductory quantum physics, modern advanced-lab experiments, and novel approaches to teach quantum mechanics and quantum optics with a laboratory component. The experiments can also serve as excellent capstone research experiences, some of which can go beyond the teaching-laboratory expectations and conclude with results that get published in research journals—all within the curricular experience.

Section III cites articles that describe single-photon experiments adapted for use in undergraduate laboratories. These include proposals for teaching the physics of light and for using light to teach quantum physics. It also includes a section on demonstrations that use classical light sources.

Admittedly, instructors who are unfamiliar with optical instrumentation will be initially challenged by the requirement of precise alignment, but such start-up problems are readily overcome. To help initiate newcomers into the experiments with single photons, Sec. IV cites articles that address technical aspects of the apparatus. The payoff is that these experiments work well, not hinging on a specific physical condition that is hard to meet. They provide conclusive results that will stimulate undergraduates.

Section V provides references to original landmark experiments that inspired the undergraduate adaptations described above. Quite often, these research articles contain thoughtful or ground-breaking interpretations of the fundamental physics underpinning the experiments. Also included are articles describing experiments or ideas that may serve as seeds for future adaptations. While undergraduates can now easily measure violations of Bell's inequalities, it is not unreasonable to imagine that future students will be teleporting a quantum state across the laboratory.

Section VI is devoted to the evolution of strategies for teaching quantum mechanics. The advent of single-photon experiments has brought a revival in the way we teach quantum mechanics. For example, it now makes sense to begin with the Stern–Gerlach experiment and linear algebra, and move to the particle-in-a-box problem to later in the course. Accordingly, Sec. VI gives a list of textbooks that have appeared that teach quantum phenomena in new ways.

Finally, Sec. VII lists web sites that contain information for those interested in more experimental details or teaching materials of the experiments. These include tutorials, price lists, alignment techniques, and laboratory write-ups.

II. JOURNALS

Advances in Atomic and Molecular Physics American Journal of Physics **Applied Optics Applied Physics Letters European Journal of Physics Europhysics Letters** Foundations of Physics Journal of Physics B Journal of the Optical Society of America B Nature Nature Communications **Optics Communications Optics Express** Physics **Physical Review** Physical Review A Physical Review Letters

Physical Review Special Topics-Physics Education Research

Physics Letters A Physics Today Progress in Optics Review of Scientific Instruments Reviews of Modern Physics Science Scientific American Scientific Reports

III. EDUCATIONAL ARTICLES

A. Implementations

This subsection presents actual implementations of photon experiments in the undergraduate setting. The articles describe several types of experiments that address fundamental concepts, such as the quantum nature of light, quantum superposition, entanglement, and nonlocality. They also contain information on equipment and inexpensive options.

- 1. "Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory," D. Dehlinger and M. W. Mitchell, Am. J. Phys. **70**, 903–910 (2002). A classic article on an experiment on photon pairs entangled in polarization for use in an undergraduate laboratory. Describes measurements that confirm polarization entanglement of photon pairs and violate a Bell inequality (Ref. 92). The companion article (Ref. 38) presents the technical details of the apparatus. (I)
- "Observing the quantum behavior of light in an undergraduate laboratory," J. J. Thorn, M. S. Neel, V. W. Donato, G. S. Bergreen, R. E. Davies, and M. Beck, Am. J. Phys. 72, 1210–1219 (2004). Presents the implementation of a fundamental experiment: How do we demonstrate the quantum nature of light? Not by the photoelectric effect or the Compton effect, but by one that shows that a photon does not split at a beam splitter. The experiment involves the measurement of the anticorrelation parameter, a recreation of the Hanbury-Brown and Twiss experiment (Ref. 60) with single photons inspired by Ref. 62. (I)
- "Interference with correlated photons: Five quantum mechanics experiments for undergraduates," E. J. Galvez, C. H. Holbrow, M. J. Pysher, J. W. Martin, N. Courtemanche, L. Heilig, and J. Spencer, Am. J. Phys. 73, 127–140 (2005). Describes the implementation of five experiments to perform in the undergraduate laboratory for a quantum mechanics course. They include quantum interference with a Mach–Zehnder interferometer, the quantum eraser, and biphoton interference, inspired by Refs. 83, 112, and 115. Reference 20 is a precursor to this article. (I)
- **4.** "Comparing quantum and classical correlations in a quantum eraser," A. Gogo, W. D. Snyder, and M. Beck, Phys. Rev. A **71**, 052103 (2005). This is an advanced undergraduate experiment that combines entanglement and quantum erasure with a polarization interferometer. Erasure is performed by the photon that does not go through the interferometer. It is similar in spirit to the experiment of Ref. 5. (I)
- "Nonlocal labeling of paths in a single-photon interferometer," M. J. Pysher, E. J. Galvez, K. Misra, K. R. Wilson, B. C. Melius, and M. Malik, Phys. Rev. A 72,

052327 (2005). This is an advanced undergraduate experiment that uses polarization entanglement and a nonpolarizing interferometer (as opposed to Ref. 4) to control the erasure by projections on the photon that does not go through the interferometer. (A)

- 6. "Quantum optics experiments with single photons for undergraduate laboratories," E. J. Galvez and M. Beck, in *Education and Training in Optics 2007*, edited by M. Nantel (SPIE Digital Library, 2007), pp. 1–8 [http://spie.org/x102158.xml]. This article presents new experimental arrangements and data for undergraduate labs with correlated photons that are complementary to Refs. 3 and 4. (I)
- "Ghost imaging: Open secrets and puzzles for undergraduates," L. Basano and P. Ottonello, Am. J. Phys. 75, 343–351 (2007). This is a general introduction to ghost imaging. Although it reports on classical experiments, it gives an introduction of the imaging found with correlated photons (Refs. 120 and 129). (I)
- 8. "Interactive screen experiments with single photons," P. Bronner, A. Strunz, C. Silberhorn, and J.-P. Meyn, Eur. J. Phys. 30, 345–353 (2009). Gives a general presentation of the single-photon source that is provided by parametric down conversion. (E)
- **9.** "Quantum mysteries tested: An experiment implementing Hardy's test of local realism," J. A. Carlson, D. M. Olmstead, and M. Beck, Am. J. Phys. **74**, 180–186 (2006). This is an adaptation of the Hardy test of local realism (Ref. 96), which is more intuitive than the conventional test (Ref. 1).(I)
- "Demonstrating quantum random with single photons," P. Bronner, A. Strunz, C. Silberhorn, and J.-P. Meyn, Eur. J. Phys. 30, 1189–1200 (2009). Presents an experiment to explore the indeterminism of quantum mechanics and entanglement. (E)
- 11. "A hands-on introduction to single photons and quantum mechanics for undergraduates," B. J. Pearson and D. P. Jackson Am. J. Phys. 78, 471–484 (2010). Reports on experiments with correlated photons for second-year physics students. It makes a thoughtful presentation for students at that level and further expands on the test on the nature of light. (I)
- 12. "Qubit quantum mechanics with correlated-photon experiments," E. J. Galvez, Am. J. Phys. 78, 511–519 (2010). Presents photon experiments in the context of a lab for a quantum-mechanics course. Supplements data and experiments of Ref. 3. (I)
- "The Hong-Ou-Mandel interferometer in the undergraduate laboratory," J. Carivioto-Lagos, P. G. Armendariz, V. Velazquez, E. Lopez-Moreno, M. Grether, and E. J. Galvez., Eur. J. Phys. 33, 1–8 (2012). This is an undergraduate adaptation of the iconic experiment of quantum optics by Mandel and coworkers (Refs. 111). (E)

See also Refs. 146 and 158 for accessible presentations of experiments in textbooks and websites, respectively.

B. Thought experiments and proposals for teaching about photons and quantum mechanics

This section contains articles that are particularly useful in presenting quantum phenomena and how students learn about it.

14. "Bringing home the atomic world: Quantum mysteries for anybody," N. D. Mermin, Am. J. Phys. 49, 940–943

(1981). Presents a simple thought experiment to illustrate Bell's inequality. (E)

- **15.** "Quantum mysteries revisited," N. D. Mermin, Am. J. Phys. **58**, 731–734 (1990). Using the same thought devices as in Ref. 14, the author explains the Greenberger–Horne–Zeilinger form of a Bell-type test for three particles. (E)
- 16. "The duality in matter and light," B.-G. Englert, M. O. Scully, and H. Walther, Sci. Am. 271, 56–61 (1994). This general article addresses the problem of quantum interference, erasure, and delayed choice. (E)
- 17. "Quantum mysteries refined," N. D. Mermin, Am. J. Phys. 62, 880–887 (1994). Following the setup of Refs. 14 and 15, the author explains the Hardy version of a Bell-type test. (E)
- "The mystery of the quantum cakes," P. G. Kwiat and L. Hardy Am. J. Phys. 68, 33–36 (2000). Gives an accessible presentation of the test of nonlocality proposed by Hardy (Ref. 96). (E)
- 19. "Multiparticle interferometry and the superposition principle," D. M. Greenberg, M. A. Horne, and A. Zeilinger, Phys. Today 46(8), 22–29 (1993). Gives a presentation on quantum interference of two and three photons. (I)
- 20. "Photon quantum mechanics and beam splitters," C. H. Holbrow, E. J. Galvez, and M.E. Parks, Am. J. Phys. 70, 260–265 (2002). A proposal for undergraduate experiments with correlated photons. (I)
- **21.** "Interaction-free measurement," A. J. DeWeerd, Am. J. Phys. **70**, 272–275 (2002). Discusses the interaction-free experiment (Ref. 78) accessible for teaching purposes. (E)
- 22. "Quantum mechanical description of linear optics," J. Skaar, J. C. Garcia Escartin, and H. Landro, Am. J. Phys. 72, 1385–1391 (2004). A presentation of the use of quantum-mechanical operators to describe linear optics experiments. (I)
- 23. "Improving students' understanding of quantum mechanics," C. Singh, M. Belloni, and W. Christian, Phys. Today 59(8), 43–49 (2006). An article on efforts to improve teaching quantum mechanics. (I)
- 24. "Interactive learning tutorials on quantum mechanics,"
 C. Singh, Am. J. Phys. 76, 400–405 (2008). A study of students' understanding of light interference and erasure. (E)
- **25.** "Entanglement, which-way measurements, and quantum erasure," C. Ferrari and B. Braunecker, Am. J. Phys. **78**, 792–795 (2010). Presents a treatment of the experiment on quantum erasure and delayed choice of Ref. 74 in a way that is accessible to undergraduates in a quantum-mechanics course. (I)
- **26.** "Teaching and understanding of quantum interpretations in modern physics courses," C. Baily and N. D. Finkelstein, Phys. Rev. Spec. Top. – Phys. Ed. Res. **6**, 010101(11) (2010). In performing fundamental experiments as teaching aids, it is important to learn about students' misconceptions and understanding. The language that instructors use can be misleading or outright incorrect. This article is an example of the type of research that also needs to be present for finding better ways to teach quantum phenomena. (E)
- 27. "Improving students' understanding of quantum mechanics via the Stern-Gerlach experiment," G. Zhu and C. Singh, Am. J. Phys. **79**, 499–507 (2011). A study

that focuses on the understanding of quantum mechanics via the Stern–Gerlach apparatus, which can be reenacted by photons going through polarizers. (I)

C. Demonstrations with classical light

This section lists experiments to illustrate quantum physics to small and large audiences. These experiments give the spirit of the quantum phenomena but use classical light sources.

- "Photon counting statistics—Undergraduate experiment," P. Koczyk, P. Wiewior, and C. Radzewicz, Am. J. Phys. 64, 240–245 (1996). Describes measurements of the statistics of photons obtained from laser and thermal light sources. (I)
- **29.** "A do-it-yourself quantum eraser," R. Hillmer and P. G. Kwiat, Sci. Am. **296**, 91–95 (2007). A popularization of the quantum-eraser experiment with simple optical components and a laser source. (E)
- **30.** "Realization of an interaction-free measurement of the presence of an object in a light beam," E. H. du Marchie van Voorthuysen, Am. J. Phys. **64**, 1504–1507 (1996). A report on an experimental demonstration to the general public of a paradoxical aspect of quantum interference and interaction-free measurement (Refs. 78 and 82). (E)
- **31.** "A simple experiment for discussion of quantum interference and which-way measurement," M. B. Schneider and I. A. LaPuma, Am. J. Phys. **70**, 266–271 (2002). With widespread use of digital cameras, it is now possible to produce demonstrations of low-light interference by the build-up of photon detections. These are great for a discussion of quantum interference. See also Refs. 33 and 34. (I)
- 32. "Quantum noise detection: A portable and educational system," J.-F. Morizur, M. Colla, and H.-A. Bachor, Am. J. Phys. 76, 1022–1025 (2008). A demonstration of properties of quantum noise. (I)
- 33. "The wave-particle duality of light: A demonstration experiment," T. L. Dimitrova and A. Weis, Am. J. Phys.
 76, 137–142 (2008). This interferometer apparatus allows students to "hear" the photon signals as the degree of interference is varied or as one of the arms is blocked. It is not a proof of the interference of single photons with themselves because the light source (a laser) is classical and thus exhibits Poissonian statistics (that is, there is finite probability that two or more photons come together even if on the average, there is less than one photon going through the interferometer). (E)
- 34. "Young's double-slit experiment with single photons and quantum eraser," W. Rueckner and J. Peidle, Am. J. Phys. 81, 951–958 (2013). Presents a modern demonstration of quantum interference with light using a high efficiency digital camera that has recently become available, which is an excellent resource for illustrating interference and the collapse of the photon wavefunction into individual camera pixels. However, as with Refs. 31 and 33, the camera measures photoelectrons and not necessarily single photon events (even if that is likely the case) because it uses a classical source, a light-emitting diode (LED). (I)

IV. LABORATORY TECHNIQUES—APPARATUS NOTES

This section lists articles on techniques for performing photon experiments. They include ones on the fundamentals

of the process; on successful experimental arrangements; and on other technical aspects such as corrections to imperfections in the optical system that degrade the fidelity of the quantum state that is produced. The articles include techniques that use both types of parametric down-conversion (type-I and type-II), with some emphasis on proposals that use the increasingly popular and inexpensive laser diode source at 405 nm. The articles explicitly present technical details and laboratory arrangements. Many articles and the websites listed in Sec. VII also give technical details within other contexts.

A. Parametric down-conversion

This subsection lists articles devoted to the process of spontaneous parametric down-conversion. Unless otherwise specified, they use the beta-barium borate (BBO) nonlinear crystal for the down-conversion process. Although all undergraduate experiments thus far use continuous-wave pump laser sources, the technology is advancing rapidly so that soon it may be possible to do demonstrations with pulsed sources. Type-I down-conversion has been the choice for implementations owing to its efficiency and conceptual simplicity, but new implementations may opt for type-II down-conversion. For example, imaging polarization entanglement with type-II down-conversion using sensitive cameras, which yields two nonconcentric but intersecting rings of photons, illustrates the mechanism of entanglement in a clear way. This section thus also includes technical aspects of type-II down-conversion.

- 35. "New high-Intensity source of polarization-entangled photon pairs," P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, Phys. Rev. Lett. 75, 4337–4341 (1995). This is the first report of the method to produce polarization-entangled photons using a type-II nonlinear crystal in which photon pairs are emitted in separate nonconcentric cones. The intersections of the cones provide directions where polarization-entangled photons are found because of the indistinguishability in the way in which they were created. This method is used widely in research investigations. A second method to produce polarization-entangled photon pairs is given in Ref. 36. (A)
- **36.** "Ultrabright source of polarization-entangled photons," P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, Phys. Rev. A **60**, 773–776 (1999). This is the first report of what has become a standard method for producing polarization-entangled states in undergraduate implementations. It involves the use of two thin orthogonally oriented nonlinear crystals, each producing a cone of photon pairs. The cones are concentric and polarized perpendicularly with respect to each other. When the cones are made to overlap, the pairs are entangled in polarization because of the indistinguishability of the crystal source. The second method is given in Ref. 35. (A)
- 37. "High-efficiency entangled photon pair collection in type-II parametric fluorescence," C. Kurtsiefer, M. Oberparleiter, and H. Weinfurter, Phys. Rev. A 64, 023802(4) (2001). Describes a technique of mode matching the down-conversion process for increased efficiency. (A)
- **38.** "Entangled photon apparatus for the undergraduate laboratory," D. Dehlinger and M. W. Mitchell, Am. J. Phys.

70, 898–902 (2002). Describes a setup for measuring entanglement and violation of Bell inequalities in an undergraduate lab. For a companion article, see Ref. 1. (I)

- 39. "Compact source of polarization-entangled photon pairs," P. Trojek, Ch. Schmid, M. Bourennane, H. Weinfurter, and Ch. Kurtsiefer, Opt. Express 12, 276–281 (2004). Reports a simple and compact source of polarization-entangled photons by type-II parametric down-conversion. (I)
- **40.** "Phase-compensated ultra-bright source of entangled photons," J. B. Altepeter, E. R. Jeffrey, and P. M. Kwiat, Opt. Express **13**, 8951–8959 (2005). Describes the production of photon pairs by the source in Ref. 36 and calculates parameters for optical elements to compensate decohering effects that degrade the fidelity of the entangled state. See also the erratum in Ref. 41. (A)
- 41. "Phase-compensated ultra-bright source of entangled photons: Erratum," G. M. Akselrod, J. B. Altepeter, E. R. Jeffrey, and P. M. Kwiat, Opt. Express 15, 5260–5261 (2007). (A)
- 42. "Generation of entangled photon pairs using small-coherence-time continuous wave pump lasers," S. Cialdi, F. Castelli, I. Boscolo, and M. G. A. Paris, Appl. Opt.
 47, 1832–1836 (2008). Provides additional discussion on compensation issues in type-I parametric downconversion. (A)
- **43.** "Collinear source of polarization-entangled photon pairs at nondegenerate wavelengths," P. Trojek and H. Weinfurter, Appl. Phys. Lett. **92**, 211103(3) (2008). Presents a collinear source of nondegenerate polarization-entangled photon pairs using type-II parametric down-conversion. (I)
- 44. "Optimizing type-I polarization-entangled photons," R. Rangarajan, M. Goggin, and P. G. Kwiat, Opt. Express 17, 18920–18933 (2009). Presents methods for optimizing the polarization entanglement by the method of Ref. 36 for both continuous-wave and pulsed sources, and a new crystal: bismuth borate (BiBO). (I)
- **45.** "Spontaneous parametric down-conversion in periodically poled KTP waveguides and bulk crystals," M. Florentino, S. M. Spillane, R. G. Beausoleil, T. D. Roberts, P. Battle, and M. W. Munro, Opt. Express **15**, 7479–7488 (2007). A complete presentation of a third very efficient source of collinear parametric down-conversion pairs using periodically polled potassium titanyl phosphate (PPKTP). It may soon be commercially available for use in teaching environments. This article represents a type of research that is currently under way to find new and efficient ways to generate photon pairs. (A)
- **46.** "Generation of polarization-entangled photon pairs in a cascade of two type-I crystals pumped by femtosecond pulses," Y. Nambu, K. Usami, Y. Tsuda, K. Matsumoto, and K. Nakamura Phys. Rev. A **66**, 033816(10) (2002). Presents a quantitative and intuitive compensation in the generation of entangled photon pairs using the source of Ref. 36 with a pulsed pump laser. (A)

B. Sources and detectors

Most techniques presented in this Resource Letter use spontaneous parametric down-conversion, which requires a short-wavelength laser whose technology is now mature, so I do not list the original articles. At the other extreme, detecting single photons, which is not practical with photomultipliers owing to their low efficiencies at near infrared wavelengths, has been limited mostly to avalanche photodiodes, which are also commercially available. Until recently, single-photon-sensitive cameras have not allowed heralding (that is, using one photon to tag the other one), so it is not known whether individual pixels record one or more photons. New triggered cameras promise to change this. The following two articles constitute a comprehensive review of sources and detectors of single photons (Ref. 47) and a research article that images the spatial mode of single photons using a triggered single-photon camera (Ref. 48).

- "Invited review article: Single-photon sources and detectors," M. D. Eisaman, J. Fan, A. Migdall, and S. Polyakov, Rev. Sci. Instrum. 82, 071101(25) (2011). (I)
- **48.** "Real-time imaging of quantum entanglement," R. Fickler, M. Krenn, R. Lapkiewicz, S. Ramelow, and A. Zeilinger, Sci. Rep. **3**, 1914 (2013). (A)

C. Electronics

Parametric down-conversion relies on coincident detection. This section lists articles that are useful in implementing low-cost coincidence circuits.

- **49.** "Coincidence counting using a field programmable gate array (FPGA)," J. W. Lord, Honors thesis (Whitman College, 2008). This resource is posted in the website of Ref. 160, which I have singled out because it gives a very useful method of coincidence detection using an inexpensive FPGA board. (I)
- "Low-cost coincidence-counting electronics for undergraduate quantum optics," D. Branning, S. Bhandari, and M. Beck, Am. J. Phys. 77, 667–670 (2009). Presents a coincidence circuit for doing undergraduate experiments. (E)
- 51. "Note: Scalable multiphoton coincidence-counting electronics," D. Branning, S. Khanal, Y. H. Shin, B. Clary, and M. Beck, Rev. Sci. Intrum. 82, 016102(3) (2011). Presents an improved FPGA-based coincidence circuit for doing multifold coincidences. (I)

D. Characterizing the quantum state and special alignments

This subsection lists articles that are helpful in characterizing the state of the light.

- 52. "Measurement of qubits," D. F. V. James, P. G. Kwiat,
 W. J. Munro, and A. G. White, Phys. Rev. A 64, 052312(15) (2001). Going beyond a Bell test, the density matrix for the entangled state can be determined by quantum state tomography. (A)
- **53.** "Quantum entanglement and the two-photon Stokes parameters," A. F. Abouraddy, A. V. Sergienko, B. E. A. Saleh, and M. C. Teich, Opt. Commun. **201**, 93–98 (2002). Presents a discussion of the two-photon Stokes parameters for characterizing the state of two polarization-entangled photons. (I)
- 54. "Characterization of the nonclassical nature of conditionally prepared single photons," A. B. U'Ren, C. Silberhorn, J. L. Ball, K. Banaszek, and I. A. Walmsley,

Phys. Rev. A **72**, 021802(4) (2005). Discusses the characterization of heralded photon sources. (A)

- **55.** "Photonic state tomography," J. B. Altepeter, E. R. Jeffrey, and P. G. Kwiat, Adv. At. Mol. Phys. **52**, 105–159 (2006). A useful in-depth tutorial on quantum tomography. (I)
- 56. "The Hong-Ou-Mandel interferometer: A new procedure for alignment," P. J. Thomas, J. Y. Cheung, C. J. Chunnilall, and M. H. Dunn, Rev. Sci. Instrum. 80, 036101(3) (2009). Describes a very useful method for setting up and aligning the Hong–Ou–Mandel interferometer. (I)

V. ORIGINAL SOURCES AND INSIGHTFUL DISCUSSIONS

This section lists articles on the original landmark experiments that inspired undergraduate implementations, discussing their motivation and significance. I also list important experiments that can serve as the basis for further demonstrations. Given the subtleties that surround quantum phenomena and the propensity for misconceptions, I also list articles that have particular relevance for a deep understanding of quantum phenomena and the nature of light.

A. Parametric down-conversion

Parametric down-conversion is central to many experimental demonstrations. The articles in this subsection offer additional insights into this process, which is discussed analytically in many other articles, in particular, those listed in Sec. IV A.

- **57.** "Theory of optical parametric noise," D. A. Kleinman, Phys. Rev. **174**, 1027–1041 (1968). This is one of the original theoretical works on spontaneous parametric down-conversion. It contains helpful discussions and derivations on the origin of the phenomenon. (A)
- 58. "Combine EPR and two-slit experiments: Interference of advanced waves," D. N. Klyshko, Phys. Lett. A 132, 299–304 (1988). An insightful view of parametric down-conversion in terms of advanced waves: the results of down-conversion can be modeled by a photon leaving one detector, being reflected at the down-conversion crystal and then reaching the other detector. (A)
- **59.** "Energy and momentum entanglement in parametric down-conversion," P. L. Saldanha and C. H. Monken, Am. J. Phys. **81**, 28–32 (2013). Presents momentum and energy entanglement in parametric down-conversion from a fundamental perspective, as due to the indistinguishability in the position and time at which the pairs were created inside the crystal. (A)

B. The photon

A discussion of quantum mechanics and complementarity leads unavoidably to photons and the nature of light. This is also a fundamental problem for teaching quantum phenomena, because light exhibits so vividly its irreconcilable wave and particle aspects. This section contains important and insightful articles on the photon.

60. "Correlation between photons in two coherent beams of light," R. Hanbury Brown and R. Q. Twiss, Nature **177**,

27–29 (1956). This landmark experiment became the birth of quantum optics. It studies the correlations in photon detections at the output ports of a beam splitter. (A)

- **61.** "The quantum theory of optical coherence," R. J. Glauber, Phys. Rev. **130**, 2529–2539 (1963). A seminal treatment of the theory of quantum optics. (A)
- **62.** "Experimental evidence for a photon anticorrelation effect on a beam splitter: A new light on single-photon interferences," P. Grangier, G. Roger, and A. Aspect, Europhys. Lett. **1**, 173–179 (1986). A fundamental article that describes a simple test of great significance: the proof that the photon exists because it does not split at a beam splitter, which inspired a number of undergraduate demonstrations (see, for example, Refs. 2 and 11). (A)
- **63.** "Beam splitting experiments with classical and with quantum particles," R. Lange, J. Brendel, E. Mohler, and W. Martienssen, Europhys. Lett. **5**, 619–622 (1988). A very instructive article on the outcomes encountered when two photons go through a beam splitter. (I)
- **64.** "Photon bunching and antibunching," M. C. Teich and B. E. A. Saleh, Prog. Opt. **XXVI**, 1–104 (1988). A thorough treatment of photon statistics. It gives useful descriptions of light sources and what defines them as classical or nonclassical. (A)
- 65. "Answer to question #45 ['What (if anything) does the photoelectric effect teach us?,' R. Q. Stanley, Am. J. Phys. 64(7), 839 (1996)]," P. W. Milonni, Am. J. Phys. 65, 11–12 (1997). An insightful comment on the (non)relevance of the photoelectric effect on the existence of photons. (E)
- **66.** "Entanglement of the orbital angular momentum states of photons," A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, Nature **412**, 313–316 (2001). This experiment involves the spatial modes of single photons and the role of the conservation of orbital angular momentum in spontaneous parametric down-conversion. Although this is a topic of much research today, the technology for making inexpensive demonstrations has not yet arrived. It emphasizes that photons carry the entire spatial mode (see also Ref. 71). (A)
- **67.** "Single-particle entanglement," S. J. van Enk, Phys. Rev. A **72**, 064306(3) (2005). This is the first of three articles (along with Refs. 68 and 69) that discuss the state of a single photon after hitting a beam splitter. They include references to other articles and controversies about the nonlocality of a single particle. (I)
- **68.** "Comment on 'single-particle' entanglement," A. Drezet, Phys. Rev. A **74**, 026301(2) (2006). (I)
- **69.** "Reply to 'Comment on "single particle" entanglement," " S. J. van Enk, Phys. Rev. A **74**, 026302(3) (2006). (I)
- **70.** "Comparing measurements of g(2)(0) performed with different coincidence detection techniques," M. Beck, J. Opt. Soc. Am. B **24**, 2972–2978 (2007). Analyzes the quantal characteristics of several types of photon sources. It is relevant to the discussions that justify the use of heralded photons in optical demonstrations of quantum interference (see, for example, Refs. 2 and 11). (A)
- 71. "Interferometric measurement of the helical mode of a single photon," E. J. Galvez, L. E. Coyle, E. Johnson, and B. J. Reschovsky, New J. Phys. 13, 053017 (2011). An experiment involving undergraduates that measures the helical mode of heralded photons. It illustrates an important aspect of the photon to emphasize to students:

photons occupy the transverse space of a spatial mode. (A)

72. "Search for patterns in sequences of single-photon polarization measurements," D. Branning, A. Katcher, W. Strange, and M. P. Silverman, J. Opt. Soc. Am. B 28, 1423–1430 (2011). An experiment that involved undergraduates in measurements of photon statistics in parametric down-conversion. (A)

C. Quantum interference and erasure

Quantum superposition and interference is at the heart of the mysteries of quantum mechanics. This section lists articles devoted to this topic that contain insightful discussions and conceptualizations.

- **73.** "Quantum eraser: A proposed photon correlation experiment concerning observation and 'delayed choice' in quantum mechanics," M. O. Scully and K. Drühl, Phys. Rev. A **25**, 2208–2213 (1982). The original proposal of the quantum eraser. (A)
- 74. "Quantum optical tests of complementarity," M. O. Scully, B.-G. Englert, and H. Walther, Nature 351, 111–116 (1991). An article on complementarity and quantum erasing. (I)
- **75.** "Observation of a nonclassical Berry's phase for the photon," P. G. Kwiat and R. Y. Chiao, Phys. Rev. Lett. **66**, 588–591 (1991). A first demonstration of photon interference while at the same time measuring the anticorrelation parameter (see Ref. 62), which measures the degree to which the light source is non-classical. In this demonstration the degree of coherence in single-photon interference is specified by the bandwidth of the detected partner photon—another form of quantum erasure. It stresses that photons are wavepackets of (coherence) length determined by the detected bandwidth. (A)
- 76. "Induced coherence and indistinguishability in optical interference," X. Y. Zou, L. J. Wang, and L. Mandel, Phys. Rev. Lett. 67, 318–321 (1991). Presents an experiment on induced coherence that boggles the mind. It is a classic experiment that can in principle be implemented with a small-scale apparatus. (A)
- 77. "Observation of a 'quantum eraser': A revival of coherence in a two-photon interference experiment," P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, Phys. Rev. A 45, 7729–7739 (1992). Presents a quantum-eraser experiment in connection with Hong–Ou–Mandel interference. (A)
- 78. "Quantum mechanical interaction-free measurements," A. Elitzur and L. Vaidman, Found. Phys. 23, 987–997 (1993). An original article on interaction-free measurement: using distinguishability in quantum interference, one can perform a nondemolition measurement (that is, measuring without disturbing). (I)
- **79.** "Optical tests of quantum mechanics," R. Y. Chiao, P. G. Kwiat, and A. M. Steinberg, Adv. At. Mol. Phys. **34**, 35–83 (1994). An insightful review of optical tests of quantum mechanics. (I)
- **80.** "Complementarity and the quantum eraser," T. J. Herzog, P. G. Kwiat, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **75**, 3034–3037 (1995). Describes a quantum-eraser experiment in a double-pass parametric down-conversion experiment. (A)

- **81.** "'Interaction-free' imaging," A. G. White, J. R. Mitchell, O. Nairz, and P. G. Kwiat, Phys. Rev. A **58**, 605–613 (1998). Presents experiments on an intriguing way to detect objects without interaction. It follows the general spirit of interaction-free measurements (Ref. 78). (I)
- **82.** "Interaction-free measurement," P. G. Kwiat, H. Weinfurter, T. Herzog, A. Zeilinger, and M. A. Kasevich, Phys. Rev. Lett. **74**, 4763–4766 (1995). This experiment implements the proposal of Ref. 78. (I)
- 83. "Quantitative wave-particle duality and nonerasing quantum erasure," P. D. D. Schwindt, P. G. Kwiat, and B.-G. Englert, Phys. Rev. A 60, 4285–4290 (1999). Describes the quantum eraser using a Mach–Zehnder interferometer, which is used as a model for undergraduate implementations (see, for example, Ref. 3). (I)
- **84.** "Quantum erasure in double-slit interferometers with which-way detectors," B.-G. Englert, M. O. Scully, and H. Walther, Am. J. Phys. **67**, 325–329 (1999). Discusses the experiment proposed in Ref. 74 in light of an errone-ous comment to that article. (I)
- 85. "Delayed 'choice' quantum eraser," Y.-H. Kim, R. Yu, S. P. Kulik, Y. Shih, and M. O. Scully, Phys. Rev. Lett.
 84, 1–5 (2000). An implementation of the original proposal of the delayed-choice quantum eraser (Ref. 73). (A)
- 86. "Double-slit quantum eraser," S. P. Walborn, M. O. Terra Cunha, S. Padua, and C. H. Monken, Phys. Rev. A 65, 033818(6) (2002). An experiment on the quantum eraser with double-slits and polarization entanglement. (I)
- **87.** "Phase shifting of an interferometer using nonlocal quantum-state correlations," E. J. Galvez, M. Malik, and B. C. Melius, Phys. Rev. A **75**, 020302(4) (2007). This experiment adds to an advanced undergraduate experiment (Ref. 5), where the phase of an interference pattern is changed by phase-shifting unitary transformations on the (entangled) photon that does not go through the interferometer. (I)
- 88. "Experimental realization of Wheeler's delayed-choice gedanken experiment," V. Jaques, E. Wu, F. Grosshans, F. Treussart, P. Grangier, A. Aspect, and J.-F. Roch, Science 315, 966–968 (2007). This experiment recreates Wheeler's delayed-choice experiment with a Mach–Zehnder type interferometer turning on or off the second beam splitter after the photon has passed the first beam splitter. (I)
- 89. "Asking photons where they have been," A. Danan, D. Farfurnik, S. Bar-Ad, and L. Vaidman, Phys. Rev. Lett. 111, 240402 (2013). This is a recent and provocative experiment on path information and interference, whereby frequency labels reveal (or not) photon paths. It also touches on a modern technique of quantum inquiry via weak measurements. (A)

D. Entanglement

This section lists articles on entanglement and fundamental tests of quantum mechanics.

90. "Can quantum-mechanical description of physical reality be considered complete?," A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777–780 (1935). One of the landmark papers on quantum mechanics of the 20th century, challenging quantum mechanics with the EPR thought experiment. (A)

- **91.** "On the Einstein-Podolsky-Rosen paradox," J. S. Bell, Physics **1**, 195–200 (1964). Bell's proposed resolution of the EPR paradox (Ref. 90), then thought to be untestable. (A)
- 92. "Proposed experiment to test local hidden-variable theories," J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880–884 (1969). This has become the standard form for doing experimental tests of Bell inequalities with light. For undergraduate implementations, see Ref. 1. (A)
- 93. "Experimental realization of the Einstein-Podolsky-Rosen-Bohm gedankenexperiment: A new violation of Bell's inequalities," A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 49, 91–94 (1982). One of the original experiments presenting a strong violation of Bell inequalities. (A)
- 94. "Experimental test of Bell's inequalities using time-varying analyzers," A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. 49, 1804–1807 (1982). Followup to Ref. 93 with the use of time-varying projections, a more stringent test of nonlocality. (A)
- 95. "Bell theorem without inequalitites," D. M. Greenberg, M. A. Horne, A. Shimony, and A. Zeilinger, Am. J. Phys. 58, 1131–1143 (1990). Describes a Bell-type test for three particles, known as the Greenberger–Horne–Zeilinger (GHZ) test. (A)
- 96. "Nonlocality for two particles without inequalities for almost all entangled states," L. Hardy, Phys. Rev. Lett. 71, 1665–1668 (1993). This is the Hardy test of nonlocality that is used as an alternative test to Ref. 92. (A)
- **97.** "Hidden variables and the two theorems of John Bell," N. D. Mermin, Rev. Mod. Phys. **65**, 803–815 (1993). A classic article that discusses fundamental tests of quantum mechanics. (I)
- 98. "Experimental demonstration of the violation of local realism without Bell inequalities," J. R. Torgerson, D. Branning, C. H. Monken, and L. Mandel, Phys. Lett. A 204, 323–328 (1995). First implementation of the Hardy test with postselected entangled states. (A)
- **99.** "Violation of Bell's inequality under strict Einstein locality conditions," G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **81**, 5039–5043 (1998). In this article the projections used in determining nonlocality are made randomly while the photons are in flight so that there cannot be any influence from one detector to the other. It closes the "locality loophole" in Bell-inequality violations. (A)
- 100. "Nonmaximally entangled states: Production, characterization, and utilization," A. G. White, D. F. V. James, P. H. Eberhard, and P. G. Kwiat, Phys. Rev. Lett. 83, 3103–3107 (1999). An implementation of the Hardy test via photon pairs in nonmaximally entangled states, an arrangement used in one of the undergraduate implementations (Ref. 9). (A)
- 101. "Experiments towards falsification of noncontextual hidden variable theories," M. Michler, H. Weinfurter, and M. Zukowski, Phys. Rev. Lett. 84, 5457–5461 (2000). This experiment performs a violation of the GHZ inequality, defined for three particles (qubits), but using two photons. It is a clever way to illustrate the

GHZ inequality with the standard source of photon pairs produced by parametric down-conversion. (A)

- **102.** "Violation of Bell's inequality with photons from independent sources," T. B. Pittman and J. D. Franson, Phys. Rev. Lett. **90**, 240401 (2003). An intriguing experiment on the violation of Bell's inequalities with photons from distinct sources but which are otherwise indistinguishable. (A)
- 103. "Realization of the Einstein-Podolsky-Rosen paradox using momentum- and position-entangled photons from spontaneous parametric down conversion," J. C. Howell, R. S. Bennink, S. J. Bentley, and R. W. Boyd, Phys. Rev. Lett. 92, 210403 (2004). This is a recrealization of the famous EPR paradox using position and momentum, as opposed to all other realizations that rely on the Bohm version of the EPR paradox. (A)
- **104.** "Do EPR correlations require a non-local interpretation of quantum mechanics? I: Wigner approach," M. O. Scully, N. Erez, and E. Fry, Phys. Lett. A **347**, 56–61 (2005). An insightful article that digresses further on the meaning of Bell's tests. (I)
- 105. "Bell-Inequality violations with single photons in momentum and polarization," B. R. Gadway, E. J. Galvez, and F. DeZela, J. Phys. B 42, 015503(9) (2009). An advanced undergraduate experiment, plus further analysis, studying the nonseparability of two modes of a single photon: momentum and polarization. (A)
- 106. "Entanglement of arbitrary superpositions of modes within two-dimensional orbital angular momentum state spaces," B. Jack, A. M. Yao, J. Leach, J. Romero, S. Franke-Arnold, D. G. Ireland, S. M. Barnett, and M. J. Padgett, Phys. Rev. A 81, 043844 (2010). Spatial modes can provide an infinite Hilbert space, and parametric down-conversion naturally provides entanglement of spatial modes. This article provides a simple demonstration of the correlations between spatial modes of two photons, which are projected by a modern spatial-mode projective device: a spatial light modulator. (A)
- 107. "Complete experimental toolbox for alignment-free quantum communication," V. D'Ambrosio, E. Nagali, S. P. Walborn, L. Aolita, S. Slussarenko, L. Marrucci, and F. Sciarrino, Nat. Commun. 3, 961 (2012). A new device, known as the q-plate, entangles spin (polarization) and orbital (spatial) angular momentum degrees of freedom, providing a simple method to encode photons in nonseparable superpositions of polarization and spatial mode. Pairs of photons can be further entangled, providing a higher quantum dimensionality. (A)
- 108. "Bell violation using entangled photons without the fair-sampling assumption," M. Giustina, A. Mech, S. Ramelow, B. Wittmann, J. Kofler, J. Beyer, A. Lita, B. Calkins, T. Gerrits, S.W. Nam, R. Ursin, and A. Zeilinger, Nature 497, 227–230 (2013). This and the next article (Ref. 109) do a Bell-violation measurement without having to invoke the fair-sampling assumption: the assumption that the reduced sample of detected correlations (due to pairs lost to detector inefficiencies) is representative of the full ensemble of photons. It closes the long-standing "detection loophole" of Bell-inequality violations. (A)
- **109.** "Detection-loophole-free test of quantum nonlocality, and applications," B. G. Christensen, K. T. McCusker, J. B. Altepeter, B. Calkins, T. Gerrits, A. E. Lita, A.

Miller, L. K. Shalm, Y. Zhang, S. W. Nam, N. Brunner, C. C. W. Lin, N. Gisin, and P. G. Kwiat, Phys. Rev. Lett. **111**, 130406 (2013). (A)

E. Two-photon interference effects

Parametric down-conversion produces pairs of photons, so it opens the door to a number of quantum-interference experiments involving two photons.

- 110. "Interference of independent photon beams," R. L. Pflegor and L. Mandel, Phys. Rev. 159, 1084–1088 (1967). This experiment unleashes a debate that helps us understand the nature of the photon and interference more deeply. I include it and the various articles that discuss the phenomenon (Refs. 124 and 125), and the controversy (Refs. 116, 117, and 118), on whether interference can arise from distinct photon sources. It does when the source of the photon is indistinguishable. (A)
- 111. "Measurement of subpicosecond time intervals between two photons by interference," C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044–2046 (1987). The iconic first report of Hong–Ou–Mandel interference. It is the basis of many photon experiments on fundamental tests and on implementing quantuminformation schemes. (A)
- 112. "Finesse and resolution enhancement in two-photon interferometry," E. Mohler, J. Brendel, R. Lange, and W. Martienssen, Europhys. Lett. 8, 511–516 (1989). This experiment adds a new twist to interference by sending two photons collinearly to an interferometer. The interference is the result of more than two amplitudes and thus produces distinct interference curves. This experiment stimulated adaptations described in Refs. 3 and 6. (A)
- 113. "Bell inequality for position and time," J. D. Franson, Phys. Rev. Lett. 62, 2205–2208 (1989). This is a very important proposal to observe and use the interference of two photons going through separate but identical interferometers. The interferometers have long and short paths, with a path difference that is longer than the coherence length of the photons going through them. However, interference appears in the indistinguishability of possibilities (short-short and long-long paths taken by both photons). (A)
- 114. "Correlated two-photon interference in a dual-beam Michelson interferometer," P. G. Kwiat, W. A. Vareka, C. K. Hong, H. Nathel, and R. Y. Chiao, Phys. Rev. A 41, 2910–2913 (1990). This is an implementation of the proposal of Ref. 113. (A)
- **115.** "Time-resolved dual-beam two-photon interferences with high visibility," J. Brendel, E. Mohler, and W. Martienssen, Phys. Rev. Lett. **66**, 1142–1145 (1991). Experiment reporting the measurement of the interference of two photons acting as one (see also Refs. 126, 127, and 128). This experiment served as inspiration to one of the undergraduate implementations (Refs. 3 and 6). (A)
- 116. "Comment on 'Interference fringes between two separate lasers,' by F. Lourador, F. Reynaud, B. Colombeau, and C. Froehly [Am. J. Phys. 61(3), 242–245 (1993)]," P. R. Wallace, Am. J. Phys. 62, 950 (1994). This is a first comment on an erroneous article

(not included here), relating to the interference from separate lasers. This comment and those Refs. 117, 118, 124, and 125 add insight to the problem. (E)

- 117. "Comment on 'Interference fringes between two separate lasers,' by F. Lourador, F. Reynaud, B. Colombeau, and C. Froehly [Am. J. Phys. 61(3), 242–245 (1993)]," L. M. Davis and C. Parigger, Am. J. Phys. 62, 951–954 (1994). The second reply to the erroneous article. (E)
- **118.** "Dirac's famous dictum on interference: One photon or two?," R. J. Glauber, Am. J. Phys. **63**, 12 (1995). Glauber, the master of quantum optics coherence, gives a third very insightful answer to the controversy over the erroneous article. (E)
- 119. "Experimental evaluation of a two-photon wave packet in type-II parametric downconversion," A. V. Sergienko, Y. H. Shih, and M. H. Rubin, J. Opt. Soc. Am. B 12, 859–862 (1995). A report of the HOM interference using collinear photon pairs, which has yet to be implemented in the undergraduate setting. (A)
- 120. "Observation of two-photon 'ghost' interference and diffraction," D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, Phys. Rev. Lett. 74, 3600–3603 (1995). A first article that discusses an experimental problem with many subtleties. It involves the detection of an object in the correlations of two beams/photons. (A)
- 121. "Can two-photon interference be considered the interference of two photons?," T. B. Pittman, D. V. Strekalov, A. Migdall, M. H. Rubin, A. V. Sergienko, and Y. H. Shih, Phys. Rev. Lett. 77, 1917–1920 (1996). This article aims at a misconception in HOM interferometry that photon wavepackets must overlap at the beam splitter to effect the indistinguishability. The authors dispel the myth by delaying one photon relative to the other before the beam splitter and undo this delay in postselection. This experiment can also be a model for a collinear HOM implementation. (A)
- **122.** "Experimental quantum teleportation," D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature **390**, 575–579 (1997). The original report on a quantum teleportation experiment. It is a type of experiment that students can follow with quantum-mechanical algebra (see, for example, Ref. 143) and yet marvel at what quantum mechanics leads researchers to conceive. I included it because as the technology improves, it may be possible to make an undergraduate implementation of it in the near future. (A)
- 123. "Experimental realization of teleporting an unknown pure quantum state via dual classical and Einstein-Podolsky-Rosen channels," D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. 80, 1121–1125 (1998). An implementation of teleportation may use the scheme in this article. (A)
- **124.** "Answer to question # 60. Interference of two independent sources," C. Kiefer, Am. J. Phys. **66**, 661–662 (1998). The answer to a question on the interference of two independent light sources. (E)
- 125. "Answer to question # 60. Interference of two independent sources," F. J. Duarte Am. J. Phys. 66, 662–663 (1998). A second answer to the question referred to in Ref. 124. (E)
- **126.** "Measurement of the de Broglie wavelength of a multiphoton wavepacket," E. J. S. Fonseca, C. H. Monken,

and S. Padua, Phys. Rev. Lett. **82**, 2868–2971 (1999). Reports on an interesting experiment where the de Broglie wavelength of a group of indistinguishable photons (two in this case; see the following two references) is the wavelength of the individual photons divided by the number of photons. (A)

- 127. "De Broglie wavelength of a non-local four-photon state," P. Walther, J.-W. Pan, M. Aspelmeyer, R. Ursin, S. Gasparoni, and A. Zeilinger, Nature 429, 158–161 (2004). This and the next article are the realization of several-photon interference experiments where the (nonclassical) fringe spacing depends on the number of photons. In this case, the authors measure the interference of four photons. (A)
- **128.** "Super-resolving phase measurements with a multiphoton entangled state," M. W. Mitchell, J. S. Lundeen, and A. M. Steinberg, Nature **429**, 161–164 (2004). Measurement of the interference of light in a three-photon state. (A)
- **129.** "Quantum and classical coincidence imaging," R. S. Bennink, S. J. Bentley, R. W. Boyd, and J. C. Howell, Phys. Rev. Lett. **92**, 033601(4) (2004). Expands on the degree to which ghost imaging is due to classical or quantal correlations. (A)
- **130.** "Generation of a two-photon singlet beam," W. A. T. Nogueira, S. P. Walborn, S. Padua, and C. H. Monken, Phys. Rev. Lett. **92**, 043602(4) (2004). Although the use of spatial modes may be beyond undergraduate demonstrations, this article is insightful in the way it manipulates the state of the light, now entangled in polarization and spatial mode, so as to preserve the bosonic symmetry of the two-photon wavefunction. (I)

VI. NEW TEXTBOOKS

The use of single photons in many recent demonstrations of quantum mechanics has led many authors to rethink the way we teach quantum mechanics. Thus, new textbooks have appeared that follow new approaches and sequences, moving entanglement and Bell's inequalities from the back of the book, the conventional approach, to the front of the book; or starting with bra-ket notation instead of the wavemechanical approach. In this section, I give a list of textbooks that provide these new approaches.

- **131.** The Feynman Lectures on Physics, Vol. 3, R. P. Feynman, R. B. Leighton, and M. Sands (Addison-Wesley, Reading, 1965) This textbook was ahead of its time in terms of introducing quantum mechanics. Modern adaptations start by using the approach of this textbook. (E)
- **132.** Introduction to Quantum Physics, A. P. French and E. F. Taylor (Norton, New York, 1978). Another classic textbook that was ahead of its time. It uses polarization beam displacers to analyze Stern–Gerlach experiments. (E)
- **133. Introductory Quantum Optics**, C. Gerry and P. Knight (Cambridge U. P., Cambridge, 2004). This textbook has discussions of photon experiments. It uses the more advanced photon-number representation. (A)
- 134. The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics, G. S. Greenstein and A. G. Zajonc, 2nd ed. (Jones and

Bartlett Publishers, Boston, 2005). This textbook has discussions on many paradigms of quantum physics. (I)

- 135. Protecting Information from Classical Error Correction to Quantum Cryptography, S. Loepp and W. K. Wooters (Cambridge U. P., Cambridge, 2006). This textbook is for nonphysics students and contains introductory presentations about quantum phenomena. (E)
- **136.** Quantum Optics, M. Fox (Oxford U. P., Oxford, 2006). This introductory textbook on quantum optics avoids the photon-number representation. (A)
- **137. Quantum Physics**, M. Le Bellac (Cambridge U. P., New York, 2006). This textbook introduces quantum mechanics using modern experiments as examples and exercises. (I)
- **138.** Entangled Systems: New Directions in Quantum Physics, J. Audretsch (Wiley-VCH, Weinheim, 2007). An advanced textbook on quantum physics geared to provide a theoretical basis for quantum information. (A)
- **139.** Quantum Reality Theory and Philosophy, J. Allday (Taylor and Francis, Boca Raton, 2009). An introductory textbook that covers the conceptual challenges of quantum physics but with no hesitation of introducing and using the mathematical formalism of quantum mechanics. (E)
- **140.** Modern Introductory Physics, C. H. Holbrow, J. N. Lloyd, J. C. Amato, E. Galvez, and M. E. Parks, 2nd ed. (Springer Verlag, New York, 2010). This is a modern physics textbook for first-year students, which contains two chapters on quantum interference and entanglement. (E)
- 141. Foundations of Quantum Mechanics—From Photons to Quantum Computers, R. Blümel (Jones and Bartlett, Sudbury, 2010). This modern modernphysics textbook starts with quantum mechanics and follows an unconventional approach. (I)
- **142.** Quantum Processes, Systems and Information, B. Schumacher and M. Westmoreland (Cambridge U. P., Cambridge, 2010). This textbook teaches and uses quantum mechanics to introduce quantum information. (A)
- **143.** A Modern Approach to Quantum Mechanics, J. S. Townsend, 2nd ed. (University Science Books, Mill Valley, 2012). This textbook starts with bra-ket notation and Stern–Gerlach apparatuses. (A)
- 144. Quantum Mechanics: A Paradigms Approach, D. H. McIntyre, C. A. Manogue, and J. Tate (Addison-Wesley, San Francisco, 2012). This textbook presents the new approach of starting with Stern–Gerlach experiments. It includes conceptual lab exercises and simulations. (A)
- **145.** Quantum Mechanics Theory and Experiment, M. Beck (Oxford U. P., Oxford, 2012). This textbook, written by a developer of photon laboratories, presents a quantum-mechanics curriculum that goes in concert with a photon-based laboratory component. (A)
- 146. Exploring Quantum Physics through Hands-On Projects, D. Prutchi and S. R. Prutchi (Wiley, Hoboken, 2012). This textbook presents a variety of laboratory demonstrations of quantum phenomena put together with inexpensive components. It contains photos, diagrams, and apparatus designs. (E)
- 147. Quantum Mechanics and Quantum Information, M. Fayngold and V. Fayngold (Wiley-VCH, Weinheim

2013). This is a comprehensive quantum-mechanics textbook that follows the new approach. (A)

VII. WEBSITES

This section lists websites that contain various types of information about undergraduate experiments, tutorials, and implementations. They are listed by institution in alphabetical order.

- 148. http://www.advlab.org/spqm.html Search phrase: "ALPhA's Single Photon Detector Initiative." Advanced Laboratory Physics Association. This website contains information about workshops to implement photon experiments. It also distributes less expensive single-photon detectors for teaching purposes. In addition, it has information on other undergraduate experiments and instructional workshops for use in the advanced laboratory component of the physics curriculum. (E)
- 149. http://labs.physics.berkeley.edu/mediawiki/index.php/ Design_and_Documentation_%28PQM%29 Search phrase: "Design and Documentation (QIE) Berkeley." University at California–Berkeley. This website has a wiki page with information about the use of photon experiments in their advanced lab course. (E)
- **150.** http://departments.colgate.edu/physics/pql.htm. Search phrase: "Photon Quantum Mechanics Colgate." Colgate University. This website contains information about experiments, procedures, lab write-ups, and price lists. (E)
- **151.** http://www.compadre.org/quantum/ Search phrase: "Compadre Quantum Exchange." ComPADRE Digital Library. This is the quantum section of an online educational resource for physics and astronomy. (E)
- **152.** http://webphysics.davidson.edu/applets/applets.html. Search phrase: "Physlets Web Physics Davidson." Davidson College. This website contains Java Applets for simulating quantum mechanics problems. (E)
- **153.** http://singlephoton.wikidot.com/welcome. Search phrase: "Photon Quantum Mechanics Wiki Dickinson." Dickinson College. This website contains a wiki page on photon experiments, including experiment information, procedures, and parts list. (E)
- 154. http://www.didaktik.physik.uni-erlangen.de/quantumlab/ english/index.html. Search phrase: "Quantumlab

Erlangen-Nuremberg." University of Erlangen-Nurenberg. This website contains interactive information about a number of photon experiments: their description, photos, interactive diagrams, and videos showing the apparatus in motion and data as it is accumulated. (E)

- **155.** http://research.physics.illinois.edu/QI/Photonics/Tomography/. Search phrase: "Quantum State Tomography Kwiat." University of Illinois. This is a very useful resource of articles, codes, and interactive calculations for doing quantum state tomography. (I)
- **156.** http://www.nist.gov/pml/div684/grp03/multicoincidence. cfm. Search phrase: "FPGA-Based Multicoincidence Recipe and Software NIST." National Institute for Standards and Technology–Maryland. This website contains information on a coincidence unit based on a field programable gate array (FPGA). (I)
- **157.** http://www.optics.rochester.edu/workgroups/lukishova/ QuantumOpticsLab/. Search phrase: "Quantum Optics, Quantum Information and Nano-optics Laboratory Rochester." University of Rochester. This website gives information on experiments developed for a quantum optics teaching laboratory. (E)
- **158.** http://blogs.scientificamerican.com/critical-opalescence/ 2013/02/08/ Search phrase: "How to Build Your Own Quantum Entanglement Experiment." Scientific American. This is the first of two blogs by George Musser on the adaptation of a quantum entanglement experiment using polarization-entangled gamma rays. The latter are produced by the electron–positron annihilation in the decay of Na²². (E)
- **159.** http://www.trincoll.edu/~dbrannin/Coincidence%20 Counting/CoincidenceHome.htm. Search phrase: "Coincidence-Counting Electronics Trinity." Trinity College. This website contains information about a coincidence counting circuit. (I)
- **160.** http://people.whitman.edu/~beckmk/QM/. Search phrase: "Modern Undergraduate Quantum Mechanics Experiments Whitman." Whitman College. This website contains information about experiments, including procedures, price lists, and methods, and downloads to implement coincidence detection units using FPGA boards. (E)

Finally, I thank Joseph C. Amato and the reviewers for their feedback and useful suggestions.