Lab 4 The Quantum Eraser

Phys434L Quantum Mechanics Lab
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1 Theory

A thorough description of the experiment is done using a formalism that combines the space of spatial directions, with the eigenstates $|\phi_x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\phi_y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and polarization, with the eigenstates $|H\rangle$ and $|V\rangle$, as shown earlier. The space of direction of propagation is two dimensional, a qubit; and the space of states of polarization is also two dimensional, another qubit. Thus we have 2 qubits. Quantum mechanics has a method to generate vectors and operator matrices of combined Hilbert spaces: it involves the tensor product, which is denoted by the symbol $\otimes$. In the tensor product operation, we multiply each element of one space (propagation direction) to each element of the other space (polarization). The ordering of spaces in the tensor product is important. In our case, we will order direction of propagation first, and polarization second.

For example, if we have a vector $|A\rangle$ in the space of propagation directions, and a vector $|B\rangle$ in the space of polarization, the tensor product of two vectors is:

$$|AB\rangle = |A\rangle \otimes |B\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ a_2 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix}.$$

The eigenstates of our experiment are then:

$$|\phi_x, H\rangle = |\phi_x\rangle \otimes |H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$|\phi_x, V\rangle = |\phi_x\rangle \otimes |V\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

Question 1 In-lab: Find the vectors for $|\phi_y, H\rangle$ and $|\phi_y, V\rangle$.

The matrices for the operators in the larger space are the tensor product of the matrices of the operators that act on each space. For example, an operator in the direction of propagation space, $\hat{A}$, combines with an operator in the polarization space, $\hat{B}$, the following way:

$$\hat{A} \otimes \hat{B} = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \otimes \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} =$$

$$\begin{pmatrix} a_1 \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} & a_2 \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \\ a_3 \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} & a_4 \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} a_1b_1 & a_1b_2 & a_2b_1 & a_2b_2 \\ a_1b_3 & a_1b_4 & a_2b_3 & a_2b_4 \\ a_3b_1 & a_3b_2 & a_4b_1 & a_4b_2 \\ a_3b_3 & a_3b_4 & a_4b_3 & a_4b_4 \end{pmatrix}$$

Notice that the ordering procedure for the elements of the matrix is the same as for elements of the vectors.

Using the tensor product we can also construct the matrices for the interferometer. The beam splitter acts on one space and not the other, so it will be the tensor product of the beam-splitter matrix (first) with the identity (second). We put identity for the polarization part because the beam splitter does not alter the polarization. The matrix for the beam splitter in the larger space will be:

$$\hat{B}_2 = \hat{B}_1 \otimes \hat{1} = \begin{pmatrix} t & r \\ r & t \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} t & 0 & r & 0 \\ 0 & t & 0 & r \\ r & 0 & t & 0 \\ 0 & r & 0 & t \end{pmatrix},$$

where $t = 1/\sqrt{2}$ and $r = i/\sqrt{2}$.

Question 2 Take-home: Verify that the beam-splitter operator $\hat{B}_2$ acting $|\phi_x, V\rangle$ does not alter the polarization of the state.

Question 3 In-lab: Find the matrix $\hat{M}_2$ for the mirrors of the interferometer: $\hat{M}_1 \otimes \hat{1}$.

Question 4 Find the matrix for the interferometer phase: $\hat{A}_2$, where δ is the phase difference between the two arms, and:

$$\hat{A}_1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}$$

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The quantum eraser has two wave plates in the arms of the interferometer. The matrix representing the half wave plate in the upper arm with angle θ and a half-wave plate with $\theta = 0$ in the lower arm is:

$$\hat{W}_2(\theta, 0) = \begin{pmatrix} \cos 2\theta & \sin 2\theta & 0 & 0 \\ \sin 2\theta & -\cos 2\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{1}$$

Question 5 Take-home: Find an expression for the full interferometer matrix (laborious): $\hat{Z} = \hat{B}_2 \hat{W}_2(\theta, 0) \hat{A}_2 \hat{M}_2 \hat{B}_2$. (You could use Matlab or Mathematica.)

Question 6 Take-home: Verify that when $\theta = \pi/4$

$$\hat{Z}_2 = \frac{1}{2} \begin{pmatrix} ie^{i\delta} & i & -e^{i\delta} & 1 \\ i & -ie^{i\delta} & 1 & e^{i\delta} \\ e^{i\delta} & -1 & ie^{i\delta} & i \\ -1 & -e^{i\delta} & i & -ie^{i\delta} \end{pmatrix}, \tag{2}$$

Question 7 Calculate the probability of photons in state $|\phi_x, V\rangle$, and entering the interferometer with the waveplate at $\theta = 0$, exit the interferometer in the same state.

Question 8 Find the final state of the light when the wave plate is rotated an angle θ.

Question 9 Find the probability of the photon being with polarization vertical, as a function of θ, given that is leaving the interferometer in the x-direction. That is $P(V/\phi_x)$

Question 10 Find the previous answer for $\theta = \pi/4$.

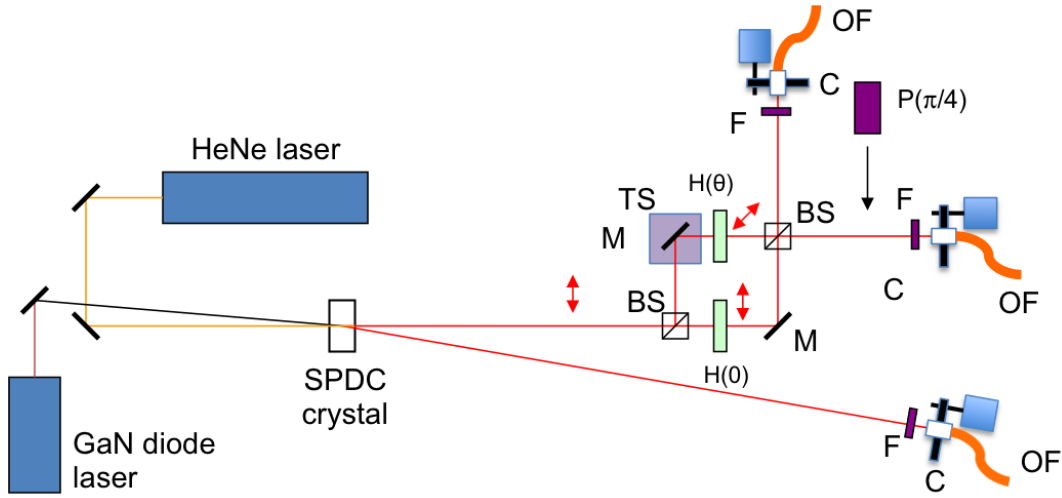**Question 11** Find $P(D/\phi_x)$ for $\theta = \pi/4$.

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Figure 1: Schematic of the setup to recreate the quantum eraser. The numbers in parenthesis refer to the numbered steps in the procedure. Additional optical elements include half-wave plates (H) and a polarizer (P).

2 The Remote Apparatus

2.1 General

This year we have to do the experiments remotely. You log into the lab computer using remote desktop. Once you do this, your screen will appear as something like what you see in Fig. 2. You will see 3 windows. One is a view of the apparatus from a webcam (upper left in the photo). The window on the lower left that is a serial interface to an Arduino based circuit that has two functions: To block/unblock the (blue) pump laser, and to turn ON/OFF the power to the detectors (more on this below). On the right is a window with the Matlab platform, with 3 programs: one rotates the half-wave plate by a specified angle, a second one places the eraser polarizer in or out of the path of the photons on their way to the Bob detector, and a third program runs the data acquisition while scanning a piezoelectric element that changes the path-length difference in the interferometer.

2.2 Commands

Here we detail the commands for the different programs. All commands given below are within quotation marks. **Do not enter the quotation marks**.

2.2.1 Laser and detectors ON/OFF

This action is for providing safety in the lab. The laser (class IV) is always on, but that creates a safety issue, so the laser beam is blocked. Turning the laser on/off does not work well because there is a warm-up time for which the laser is unstable and its wavelength
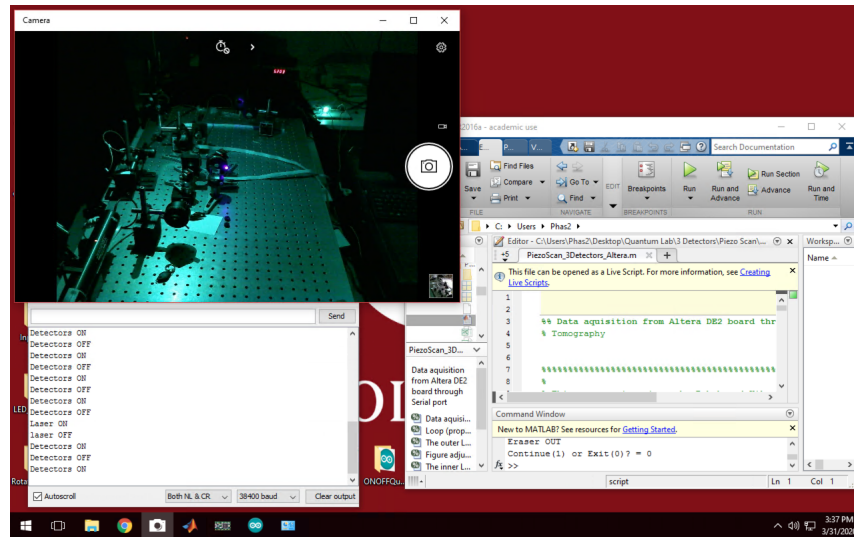
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Figure 2: Screen of the Desktop that controls the experiment in the lab. It has a webcam window (upper-left), a serial interface window (lower left) and a Matlab window (right).

drifts. The laser blocker and detector power are controlled by a program that runs on an Arduino/Teensy circuit (in case you know what this is). We have this option for the 3 detectors (Alice, Bob and Charlie) because they are very sensitive and so it is better to turn them off when not in use.

The Arduino/Teensy program is always running. A serial window is shown in the figure. The command is entered on the top command line. The action taken is listed in the lower part of the window. A summary of commands is:

- **Laser ON (Unblocked)**: On the command line type "1" . Turn it on before you start the experiments. You can do two visual verifications that this action is in place: there is a bright blue LED in the upper right part of the webcam window indicating that power is being applied to the device that unblocks the beam (with no power the beam is blocked). The beam also hits a beam block, so you can actually see the beam hitting the beam block.

- **Laser OFF (Blocked)**: On the command line type "2" . Turn it off once you are done with the lab session.

- **Detectors ON**: On the command line type "3" . Turn it on before you start the experiments. There is a green LED that is turned on when the detectors are connected to the power supply. There is also a red digital readout near the top-central part of the webcam window that indicates how much voltage is being applied to the detectors. This voltage should be close to 5 V. The value should be slightly above 5 V but never below 4.5 V (otherwise the detector signals will be unstable).

- **Detectors OFF**: On the command line type "4" . Turn it off once you are done

with the lab session.

- **Cold Restart**: In case you need to do this, in the command bar there is a round icon for the Arduino platform. You click on this. Then you click a right arrow in the upper left that compiles the program and loads it to the circuit. You then need to go to the top menu bar, click "Tools" and then click "Serial Monitor." This opens the screen that you see in the figure. Resize it and put it in the lower left. You can then minimize the Arduino platform.

2.2.2 Half-wave Plate Rotation (Matlab)

In the Matlab command window (lower part) enter "Eraser_Waveplate" . When the program runs, all actions are entered in the same command line. The commands are:

- **COM port** You get prompted for this. Enter "8" .

- **Enter Half-wave Plate Angle**: You get prompted to type an angle in degrees. So you enter the number (e.g., 30) and hit return. When you do this you can actually see the waveplate rotating in the webcam screen. The waveplate in question is as shown in a closeup of the interferometer in Fig. 3.
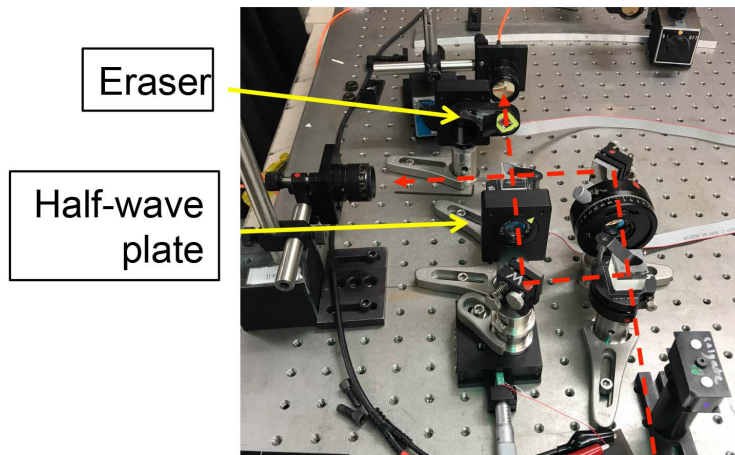

Figure 3: Photo of the interferometer showing the path of the light and the two active elements: a rotating half-wave plate and a shifting in/out eraser polarizer.

2.2.3 Eraser Polarizer in/out

In the Matlab command window (lower part) enter "Eraser_Polarizer" . When the program runs, all actions are entered in the same command line. The commands are:

- **COM port** You get prompted for this. Enter "12" .

- **Eraser IN**: Enter "i" to get the eraser placed in the path of the beam. The optical device is a polarizing beam splitter. It is attached to a rotating mount that rotates the polarizer into place. The command window will display the "IN" status of the eraser.

- **Eraser OUT**: Enter "o" to get the eraser placed out of the path of the beam. The rotating mount that rotates the polarizer out of the path of the beam. The command window will display the "OUT" status of the eraser.

2.2.4 Piezo Scan

This is the standard Matlab program to do scans. Go to the directory: "Desktop¿QuantumLab¿3 Detectors¿Piezo Scan", and run "PiezoScan_3Detectors_Altera" . A new screen will appear with several entries:

- **COM Port of Counter**: Enter "COM11"

- **COM Port of Arduino**: Enter "COM7"

- **Start Voltage**: Enter "1"

- **End Voltage**: We recommend "8", but you can use a smaller value if you want a short scan. The maximum value is 9.

- **V Increment**: Use "0.05"

- **Time Interval**: We recommend "1", but you can use more for more counts per point or less for a fast scan.

- **Excel File Name**: Here enter your own file name, something that identifies the group and the specific scan that you took. We recommend that you write in a log book the details of each scan and just label the files by sequence. The word "Eraser" will be appended to the file name. When the scan is done it will place all the data in this file, in the directory where the program resides.

3 Procedure

1. Using the instructions above unblock the laser, turn the detectors ON. Make sure that they are as they should via the webcam.

2. Run "Eraser_Waveplate" so you get familiar with it. Enter an angle between less than 360 and view the waveplate rotate. Set it to 0 when you are done.

3. Run "Eraser_Polarizer" to get familiar with how it works, also using the webcam to verify it is working.

4. Run the Piezo scan program. Take a scan to obtain the interference of the photons going through the interferometer. (Coincidences AB and AC.) Record the maximum and minimum counts of interference in the coincidences in you log book.

5. Now rotate one of the half-waveplate to 15°and repeat the scan (you will have to flip programs). Do you see any change from before? Compare with the previous maxima and minima.

6. Repeat for the half-wave plate at 30°.

7. Finally rotate the half-waveplate to 45°. Because the new state after the rotated half-waveplate is now $|H\rangle$, it is orthogonal to the state of the light coming through the other arm. The paths are distinguishable and therefore we should not get any interference. Apparatus imperfections may not lead to exactly zero oscillations. Compare this with the result of the previous question and with your calculations.

   Notice that we don't even try to measure the polarization of the paths. Interference nearly disappears even if *in principle* we can obtain the path information.

8. Now activate the program to place the polarizer (that projects the light to state $|D\rangle$) in the path of the light going towards B. This is "Eraser_Polarizer"

Question 12 Find the vector for the final state, $|\phi_x, D\rangle$.

We can account for the eraser in the quantum mechanics algebra: we create a matrix that represents the eraser operation. It must project the polarization along the x-direction after the interferometer onto the state $|D\rangle$, or

$$\hat{P}_D = |D\rangle\langle D|. \tag{3}$$

In the y-direction there is no eraser, so the matrix in the combined spaces is

$$\hat{E}_{xy} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

Notice that in the top-left we have the matrix for the projection along $|D\rangle$, to effect the projection in the x-subspace; and in the lower-right we have the identity matrix so that the polarization along the y-direction is left untouched. The operator for interferometer plus eraser will then be

$$\hat{E}_{xy}\hat{Z}. \tag{5}$$

Question 13 Now find the probability of detecting the photon after the polarizer. That is, we want to find $P(D/\phi_x)$ for the state after the eraser.

9. Take a scan and compare the results with your predictions.

10. Based on the above analysis comment on the coincidences AC.

The last part is called the "quantum eraser." By placing the polarizer after the interferometer we erase the path-labeling information, and thus we regain interference. It is striking that we decide whether to get or not the interference *after* the photon goes through the interferometer.