

Lab 3 Single-Photon Interference and the Existence of the Photon- Part 2: Experiments

Phys434L Quantum Mechanics Lab
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1 Experiment 1: Quantum Superposition

1.1 Theory

We use a Mach-Zehnder interferometer. The interferometer has two 50-50 beam splitters and two mirrors. Consider the interferometer shown in the Fig. 1. In such an interferometer,

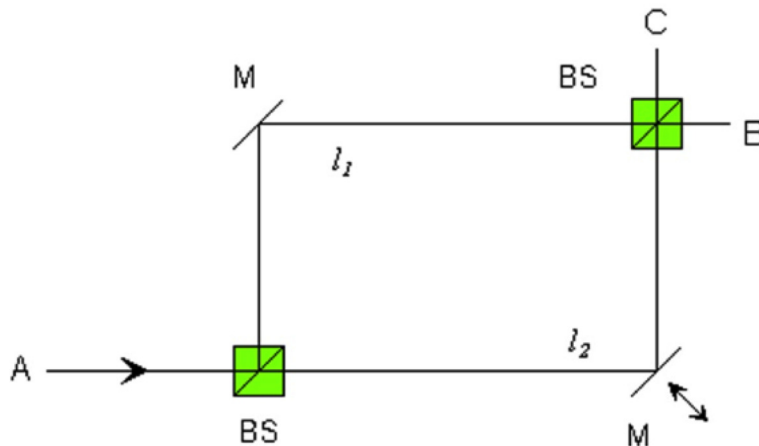


Figure 1: A Mach-Zehnder interferometer.

what varies is the direction of the momentum of the photons: they could go either in the x or y directions. We denote the state of the photon traveling along the x and y directions by $|\phi_x\rangle$ and $|\phi_y\rangle$, respectively. Suppose the a photon with a normalized wave-function $|\phi_0\rangle = |\phi_x\rangle$ is incident on the interferometer from A traveling along the x -direction. Eventually the photon will exit through either port B or port C. Thus, the final wavefunction past the

second beam splitter will be

$$|\phi'_0\rangle = b|\phi_x\rangle + c|\phi_y\rangle. \quad (1)$$

The complex coefficients b and c are the probability amplitudes for the photon going through ports B and C , respectively. Normalization thus implies that

$$b^*b + c^*c = 1. \quad (2)$$

To construct expressions for b and c we need to consider how the photon gets to B and C . Since the optical components of the interferometer affect the light as it goes through them, then quantum mechanically we will represent them by operators. The beam-splitters either reflect or transmit a photon. The operators for a beam-splitter applied to the state vectors are

$$\hat{B}|\phi_x\rangle = r|\phi_y\rangle + t|\phi_x\rangle \quad (3)$$

$$\hat{B}|\phi_y\rangle = t|\phi_y\rangle + r|\phi_x\rangle. \quad (4)$$

where $r = i/\sqrt{2}$ and $t = 1/\sqrt{2}$. Notice that the probabilities for being reflected or transmitted are $1/2$, and that reflection carries a phase of $\pi/2$.

Question 1 If the matrix representation of the state vectors is

$$|\phi_x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (5)$$

and

$$|\phi_y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (6)$$

what is the matrix representing \hat{B} ?

The mirrors change the direction of the photon, or swap the eigenstates, so they are represented by the operator \hat{M}

$$\hat{M}|\phi_x\rangle = |\phi_y\rangle \quad (7)$$

$$\hat{M}|\phi_y\rangle = |\phi_x\rangle. \quad (8)$$

Question 2 Construct the matrix representing \hat{M} .

Because the arms of the interferometer have in general different lengths, we will create an operator for the propagation of the light through each arm of the interferometer. The “arm-propagating” operator will be:

$$\hat{A} = \begin{pmatrix} e^{i2\pi\ell_1/\lambda} & 0 \\ 0 & e^{i2\pi\ell_2/\lambda} \end{pmatrix}, \quad (9)$$

where ℓ_i is the length of distance traveled in arm i . The arm operator is unitary, so the state vectors are eigenstates of the arm-propagating operator.

The State of the light after the interferometer is

$$|\phi'_0\rangle = \hat{Z}|\phi_0\rangle \quad (10)$$

where

$$\hat{Z} = \hat{B}\hat{A}\hat{M}\hat{B} \quad (11)$$

Question 3 Find an expression for \hat{Z} (Mathematica is ideal for this).

Question 4 What is the wave-function of the photon past the second beam splitter? (Use matrix notation.)

What does this mean?

We start with the photon in a state. The first beam splitter puts the photon in a superposition of states. The second beam splitter puts the state of the light also into a superposition of output states. When we do a measurement the probability amplitude is not a constant number: the total probability amplitude for going to B is the superposition of the probability amplitudes for going through both arms. The probability of going to B shows *interference*. The interesting part of this is that in quantum mechanics superposition means that the path that the photon takes is undefined. If the photon is one whole thing how could that be? Note that this is not particular to photons. One could do superposition of electrons, atoms, etc. Scientists in Vienna have done interference experiments with buckyballs.

Question 5 Find simplified expressions for b and c (see Eq. 1.)

When we put a detector in the direction x after the interferometer we are effectively applying the projector operator associated to path x :

$$\hat{P}_X = |\phi_x\rangle\langle\phi_x|. \quad (12)$$

Question 6 Find the matrix for \hat{P}_X .

Question 7 Show that the probability that the photon be at B is

$$P_B = b^*b = \frac{1}{2}(1 + \cos \delta), \quad (13)$$

where $\delta = 2\pi(\ell_1 - \ell_2)/\lambda$.

Question 8 What is the probability of the photon being in C after going through the interferometer? Consider the conservation of probability.

In summary, a consequence of superposition is *quantum interference*. By offering alternative paths, the interferometer naturally produces superposition. There is something different about the two paths: they could have different length. Thus if we vary the difference in lengths, we vary δ , and therefore the probability is going to oscillate between 0 and 1. In the lab we will change δ by changing ℓ_2 . We do this by pushing one of the mirrors very slightly (i.e., a few optical periods in very small increments). We accomplish this by means of a piezo-electric ceramic, which expands when a voltage is applied to it. The computer sends the voltage to the piezo in a stepwise manner so that we can scan δ .

1.2 Apparatus

The apparatus that we will use is shown in Fig. 2.

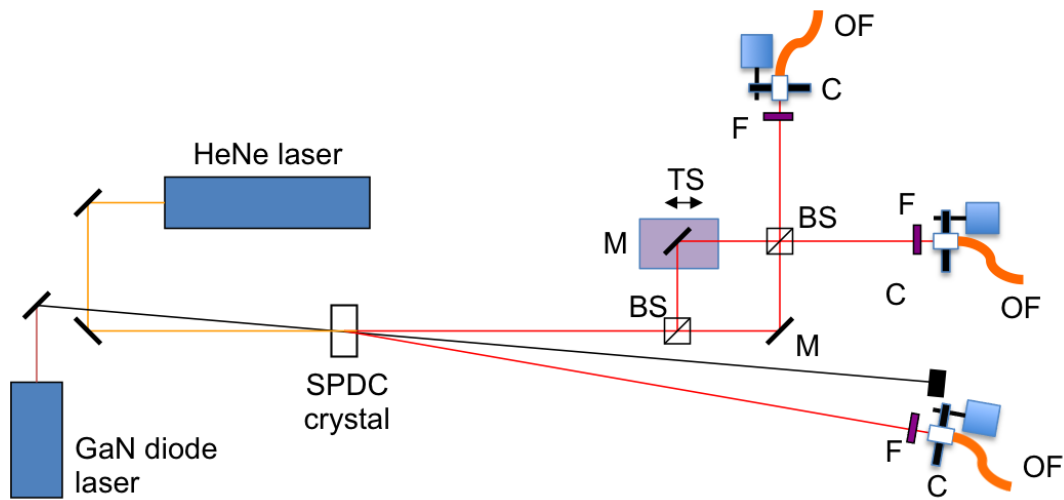


Figure 2: Apparatus to be used in the experiments. Optical elements of the interferometer are: beam splitters (BS), mirrors (M), with one of them mounted on a translation stage (TS). The photon-collection system has a 40-nm band-pass filter (F), a lens-collimator (C), and an optical fiber (OF). Photons are sent throughout the fiber to the detectors.

1.2.1 Remote Actions

We have several remote controls at our disposal:

- We can block/unblock the pump laser via the Arduino program.
- We can turn on/off the power to the detectors via the Arduino program.

- We can block/unblock the alignment laser so you can see the path of the down-converted photons heading to the polarization analyzer. We can only do this while the detectors are off. We do this via the Arduino program.
- We can block or unblock one of the arms of the interferometer.
- We record the photons that are detected by Alice, Bob and Charlie. We can also record the coincidences between Alice and Bob and between Alice and Charlie. We do this with a Matlab program.

1.2.2 Summary of Commands

- **Pump Laser ON (Unblocked):** On the command line of the Arduino platform type “1” . Turn it on before you start the experiments. You can do two visual verification that the beam is hitting the polarizer mount.
- **Pump Laser OFF (Blocked):** On the command line of the Arduino platform type “2” . Turn it off once you are done with the lab session.
- **Detectors ON:** On the command line of the Arduino platform type “3” . Turn it on before you start the experiments. This voltage should be close to 5 V. The value should be slightly above 5 V but never below 4.5 V (otherwise the detector signals will be unstable).
- **Detectors OFF:** On the command line of the Arduino platform type “4” . Turn it off once you are done with the lab session.
- **Alignment Laser ON (unblocked):** On the command line of the Arduino platform type “5”. **Before you do this you must be sure that the detectors are off:** the digital display seen with the webcam must read 0.000.
- **Alignment laser OFF (blocked):** On the command line of the Arduino platform type “6”.
- **Cold Restart:** In case you need to do this, in the command bar there is a round icon for the Arduino platform. You click on this. Then you click a right arrow in the upper left that compiles the program and loads it to the circuit. You then need to go to the top menu bar, click “Tools” and then click “Serial Monitor.” This opens the screen that you see in the figure. Resize it and put it in the lower left. You can then minimize the Arduino platform.
- **Menu** On the command line of the Arduino platform type “0” for a display of the menu of commands.
- **Block Arm:** To block one of the arms of the interferometer we use the serial communications platform “Blocker” and enter “ma810”

- **Unblock Arm**To unblock the arm enter “ma0” in the serial communications platform “Blocker.”

1.3 Procedure

The apparatus should be aligned from the previous lab. Once we are ready to detect single photons we unblock the laser on and then we power the detectors. Your tasks are the following:

1. Use the program “PiezoScan_3Detectors_Altera” take a scan. It will show the counts at detectors A B and C, and the doubles in the coincidences AB, AC and the triples ABC. Once you are satisfied that it is all working, take a long scan of, say 3 s or more per point.

Question 9 Compare the graphs of AB, AC and reflect on the physics of the phenomenon you are observing. For the report show the two in a single graph. Remember to include error bars (\sqrt{N}).

2. Block one of the arms and observe the signals. Comment on your observations? Note that the apparatus may show some imperfections. If the beam splitters are not ideal, then $t \neq 1/\sqrt{2}$.

2 Experiment 2: Existence of the Photon

2.1 Theory: Are Photons Whole?

This experiment holds even more mystery. We see that single photons interfere with themselves. How do we know the “themselves” part? The photons going through the interferometer end up in detectors Bob and Charlie. Either one of them, ...or both? We could investigate this by looking at the detections by Charlie and Bob. If both detect together, then the detectors are detecting fractions of a photon (among other implications). If the photons are whole, then they should be detected by either Charlie or Bob, but not both. We have the capability of verifying this because our apparatus can measure *triple* coincidences (of Alice, Bob and Charlie).

To be more rigorous we follow an experiment made in the 1950’s by Hanbury-Brown and Twiss. It entails a simple setup: a photon reaching a beam splitter, with detectors at both outputs. It involves measuring an anticorrelation parameter, also known as the degree of second-order coherence, $g_2(0)$. If we have detectors Bob (B) and Charlie (C) at the two outputs of the interferometer, then

$$g_2(0) = \frac{\mathcal{P}_{BC}}{\mathcal{P}_B \mathcal{P}_C}, \quad (14)$$

where \mathcal{P}_B and \mathcal{P}_C are the probabilities of detection at detectors B and C , respectively, and \mathcal{P}_{BC} is the probability of detecting photons in both detectors simultaneously. If the photon is whole, then $\mathcal{P}_{BC} = 0$, and so $g_2(0) = 0$. If the beam splitter is 50-50 and photon were to split like a classical wave, then $\mathcal{P}_{BC} = 1$, and $g_2(0) = 1$. There is a more rigorous analysis of this that we defer to the interested reader.¹

In our setup we account for the probabilities in the following way. If the heralding photon is recorded at detector Alice (A), then based on the detections at detectors A, B and C,

$$\mathcal{P}_B = \frac{N_{AB}}{N_A} \quad (15)$$

$$\mathcal{P}_C = \frac{N_{AC}}{N_A} \quad (16)$$

$$\mathcal{P}_{BC} = \frac{N_{ABC}}{N_A}. \quad (17)$$

Note that the last equation involves triple coincidences. The degree of second-order coherence then becomes

$$g_2(0) = \frac{N_{ABC}N_A}{N_{AB}N_{AC}}. \quad (18)$$

We can estimate the uncertainty based on simple propagation of Poissonian statistical errors in the counts:

$$\Delta g_2(0) = g_2(0) \left(\frac{1}{\sqrt{N_{ABC}}} + \frac{1}{\sqrt{N_{AB}}} + \frac{1}{\sqrt{N_{AC}}} + \frac{1}{\sqrt{N_A}} \right) \quad (19)$$

This type of experiment was first conducted by Hanbury Brown and Twiss in 1956. And we know the answer: the photon exists.

We will repeat the experiment here but with an even more striking beam splitter: the Mach-Zehnder interferometer! What is intriguing is the possibility that the photon somehow splits while going through the interferometer, like a wave, to interfere, but is detected whole in either output, like a particle. It is overwhelming just to think about it, but here, we will see it. Moreover, it is quite intriguing that if you think about it, right before detection there is only probability amplitude, which perhaps extends all the way to right before the photon is reaching both detectors, but when the detection is made, the photon pops in only one place.

2.2 Procedure

1. Go to the directory ‘3 Detectors/Piezo_Scan’ and use the program ‘PiezoScan_3Detectors_Altera’ take a fast scan. It will show the counts at detectors A B and C, and the doubles in the coincidences AB, AC and the triples ABC. Once you are satisfied that it is all working, take a long scan of, say 3 s or more per point.

¹J.J. Thorn et al. *American Journal of Physics* 72, 1210-1219 (2004).

Question 10 Compare the graphs of AB, AC and reflect on the physics of the phenomenon you are observing. For the report show the two in a single graph. Remember to include error bars.

2. Do a scan that shows just 4 points along the interference pattern (pick a suitable starting voltage, ending voltage and increment), and then make the time interval to be 60s. For each recorded point,
3. Find the value of $g_2(0)$ from your data and Eq. 18.
4. Calculate the uncertainty $\Delta g_2(0)$ from Eq. 19.
5. Discuss the results: by how many standard deviations is $g_2(0) < 1$? State your conclusions and any thoughts on what it all means.

3 Epilogue

Contrary to common belief, the photoelectric effect and even the Compton effect do not constitute proofs that photons exist because they can be explained using classical waves and quantum detectors. Photons by their definition must be detected as whole items. In the experiment with a beam splitter (or more dramatically, the interferometer) we detect the whole photon either on the transmitted output or on the reflected one, but not on both. It constitutes a proof that the photon exists.