Lab 4 The Quantum Eraser

Phys434L Quantum Mechanics Lab
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The goal of the lab is to investigate interference further and to apply the quantum formalism to two-Hilbert spaces.

1 Theory

1.1 A Simple Description

First let us understand the physics with simple arguments and distinguish the forest from the trees so to speak. The beam splitters have reflection and transmission amplitudes r and t, respectively. We could assume that the beam splitters are identical, but that is not necessarily the case. The probability amplitude of reaching detector B through one arm is rt, and the probability via the other arm is also rt. Because the arms lengths may not be exactly the same then there is a phase difference between the two arms $\delta = 2\pi\Delta L/\lambda$, where ΔL is the difference in length of the two arms and λ is the wavelength. so the total probability amplitude, Feynman's style is the sum of the probability amplitudes for the two indistinguishable paths:

$$p_B = rt + rte^{i\delta}. \tag{1}$$

The probability is $P_B = |p_B|^2$. Likewise, the probability amplitude in getting to C is rr through the top arm (reflection-reflection) and tt via the lower arm (transmission-transmission). There is still a phase due to the path length difference. Thus, the probability amplitude in getting to C is:

$$p_C = rr + tte^{i\delta}, \tag{2}$$

with the probability then being $P_C = |p_C|^2$. If $t = 1/\sqrt{2}$ and $r = i/\sqrt{2}$, then you get the familiar expressions for interference in the probabilities.

Question 1 By evaluation of the above expressions, show that $P_B + P_C = 1$. Do this calculation by hand.

If the paths are distinguishable then the probabilities add separately:

$$P_B = |rt|^2 + |rte^{i\delta}|^2 = 1/2 \tag{3}$$

and

$$P_C = |rr|^2 + |tte^{i\delta}|^2 = 1/2. \tag{4}$$

In this experiment the paths become distinguishable due to polarization: we rotate the polarization of the photon as it is traveling through one of the arms.

Then comes the eraser. After the interferometer in path B we put a diagonal polarizer, which has a probability amplitude of $q = 1/\sqrt{2}$ of transmitting vertical or horizontal polarizations. After the polarizer both photons have the same polarization (diagonal) and the path information is erased. The paths are indistinguishable again. The probability amplitude in getting to B is now

$$p'_B = rtq + rtqe^{i\delta} \tag{5}$$

Question 2 Find the probability $P'_B = |p'_B|^2$.

The last part is called the "quantum eraser." By placing the polarizer after the interferometer we erase the path-labeling information, and thus we regain interference. It is striking that we decide whether to get or not the interference *after* the photon goes through the interferometer.

1.2   Distinguishability

Now we do a more rigorous analysis using the quantum mechanics algebra. In this experiment we have two degrees of freedom of a single particle, so the description combines the space of spatial directions, with the eigenstates $|X\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|Y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and polarization, with the eigenstates $|H\rangle$ and $|V\rangle$, as shown earlier. The space of direction of propagation is two dimensional, and the space of states of polarization is also two dimensional. Quantum mechanics has a method to generate vectors and operator matrices of combined Hilbert spaces: it involves the tensor product, which is denoted by the symbol $\otimes$. In the tensor product operation, we multiply each element of one space (propagation direction) to each element of the other space (polarization). The ordering of spaces in the tensor product is important. In our case, we will order direction of propagation first, and polarization second.

The eigenstates of our experiment are then:

$$|XH\rangle = |X\rangle \otimes |H\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{6}$$

$$|XV\rangle = |X\rangle \otimes |V\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \tag{7}$$

$$|YH\rangle = |Y\rangle \otimes |H\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \tag{8}$$

$$|YV\rangle = |Y\rangle \otimes |V\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{9}$$

Using the tensor product we can also construct the matrices for the interferometer. The non-polarizing beam splitter acts on one space and not the other, so it will be the tensor product of the beam-splitter matrix (first) with the identity (second). We put identity for the polarization part because the beam splitter does not alter the polarization. The matrix for the beam splitter in the larger space will be:

$$\hat{B}_2 = \hat{B}_1 \otimes \hat{1} = \begin{pmatrix} t & r \\ r & t \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} t & 0 & r & 0 \\ 0 & t & 0 & r \\ r & 0 & t & 0 \\ 0 & r & 0 & t \end{pmatrix},$$

where $t = 1/\sqrt{2}$ and $r = i/\sqrt{2}$.

Question 3 Find the matrix $\hat{M}_2$ for the mirrors of the interferometer.

The matrix for the interferometer phase: $\hat{A}_2 = \hat{A}_1 \otimes \hat{1}$:

$$\hat{A}_1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \tag{10}$$

where δ is the phase difference between the two arms.

Question 4 Find the matrix $\hat{A}_2$ for the interferometer phase.

The quantum eraser has two wave plates in the arms of the interferometer. The matrix representing the half wave plate in one arm with angle θ and a half-wave plate with $\theta = 0$ in the other arm is:

$$\hat{W}_2(\theta, 0) = \begin{pmatrix} \cos 2\theta & \sin 2\theta & 0 & 0 \\ \sin 2\theta & -\cos 2\theta & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{11}$$

Question 5 Find an expression for the full interferometer matrix (laborious):

$$\hat{Z} = \hat{B}_2 \hat{A}_2 \hat{M}_2 \hat{W}_2(\theta, 0) \hat{B}_2. \tag{12}$$

(You should use Mathematica. By the way, tensor product of $\hat{A}$ and $\hat{B}$ is KroneckerProduct$[A, B]$). Keep the answer in terms of r and t.

3

Question 6 Take-home: Verify that when $\theta = \pi/4$

$$
\hat{Z}_2 = \begin{pmatrix}
-rt & rte^{i\delta} & -t^2 & r^2 e^{i\delta} \\
rte^{i\delta} & rt & r^2 e^{i\delta} & t^2 \\
-r^2 & t^2 e^{i\delta} & -rt & rte^{i\delta} \\
t^2 e^{i\delta} & r^2 & rte^{i\delta} & rt
\end{pmatrix},
\tag{13}
$$

The initial state is the photon traveling along x vertically polarized, or in state $|XV\rangle$. If we measure only on the output along the x-axis then the operator for the photons heading in that direction involves projecting the state past the interferometer:

$$
\hat{P}_X = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\tag{14}
$$

The final state will be a superposition of $|XH\rangle$ and $|XV\rangle$, so the final probability of detecting a photon will be

$$
P(\delta) = |\langle XH|\hat{P}_X \hat{Z}_2|XV\rangle|^2 + |\langle XV|\hat{P}_X \hat{Z}_2|XV\rangle|^2.
\tag{15}
$$

Question 7 Find the vector for $\hat{P}_X \hat{Z}_2|XV\rangle$ (Mathematica is fine).

Question 8 Find the probability $P(\delta)$ when $\theta = 0$. You must do this by hand, because Mathematica gets complicated. Replace $t = 1/\sqrt{2}$ and $r = i/\sqrt{2}$. It should look familiar.

We can derive that for an angle θ

$$
P(\delta) = \frac{1}{2}\left(1 + V\cos\delta\right),
\tag{16}
$$

where

$$
V = \cos 2\theta
\tag{17}
$$

is called the visibility of the interference fringes. Note that θ goes from 0 to $\pi/4$, or equivalently, V from 1 to 0, respectively.

The previous equations are very significant. When $\theta = 0$, $V = 1$ and the probability is $P = (1/2)(1 + \cos\delta)$, showing interference, a case where the paths are indistinguishable. When $\theta = \pi/4$ we get $V = 0$, and so $P = 1/2$ and there is no interference. But that approach cannot get us to all cases. A case when the paths are fully distinguishable by polarization. Note that this is consistent with the more intuitive approach of Feynman, but that approach does not easily give results for all the cases allowed by Eq. 17.

The distinguishability of the paths D is given by

$$
D = 1 - V.
\tag{18}
$$

4

In the experiment we measure photon counts, which are proportional to the probability:

$$N = N_0 P(\delta), \tag{19}$$

When $\delta = 2n\pi$ (n integer) we get the maximum photon counts

$$N_{\text{max}} = \frac{N_0}{2}(1 + V), \tag{20}$$

and when $\delta = m\pi$ (m odd integer) we get the minimum counts:

$$N_{\text{min}} = \frac{N_0}{2}(1 - V). \tag{21}$$

Since we do not know N_0, we can use Eqs. 20 and 21 to get the visibility from measurable quantities:

$$V = \frac{N_{\text{max}} - N_{\text{min}}}{N_{\text{max}} + N_{\text{min}}}. \tag{22}$$

1.3 Erasure

We then add the eraser: a polarizer with transmission axis along the diagonal direction. We put his element in the x-output of the interferometer. We can account for the eraser in the quantum mechanics algebra: we create a matrix that represents the eraser operation. It must project the polarization along the x-direction after the interferometer onto the state $|D\rangle$, or

$$\hat{P}_D = |D\rangle\langle D|. \tag{23}$$

In the y-direction there is no eraser, so the matrix in the combined spaces is

$$\hat{E}_{xy} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{24}$$

Notice that in the top-left we have the matrix for the projection along $|D\rangle$, to effect the projection in the x-subspace; and in the lower-right we have the identity matrix so that the polarization along the y-direction is left untouched. The operator for interferometer plus eraser will then be

$$\hat{E}_{xy}\hat{Z}. \tag{25}$$

Question 9 Show

$$|\langle XD|\hat{E}_{xy}\hat{Z}|XV\rangle|^2 = \frac{1}{4}(1 + \cos\delta). \tag{26}$$
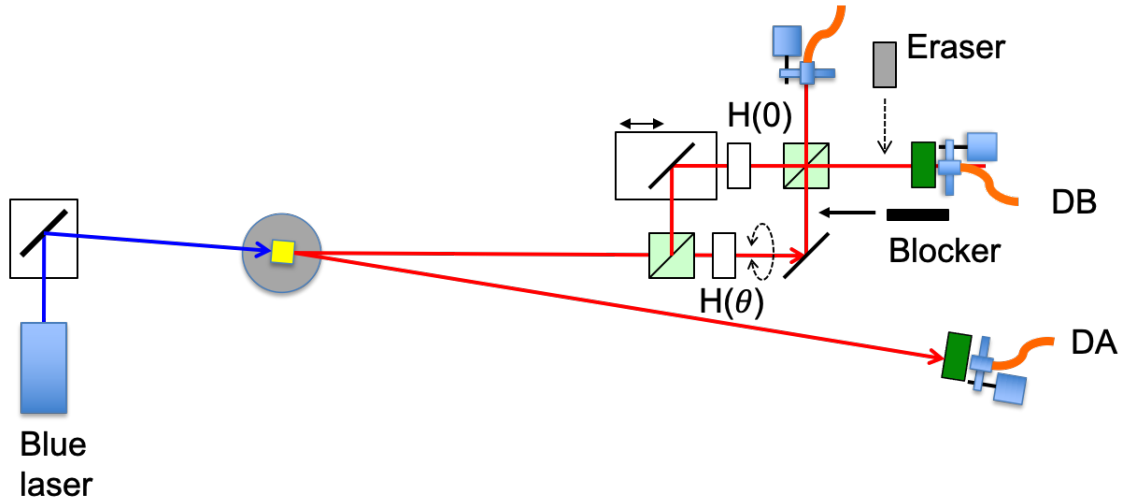
Figure 1: Schematic of the setup to recreate the quantum eraser. The numbers in parenthesis refer to the numbered steps in the procedure. Additional optical elements include half-wave plates (H) and a polarizer (P).

2 The Remote Apparatus

2.1 General

You log into the lab computer using remote desktop. Once you do this, your screen will appear as something like what you see in Fig. 2. You will see various windows. Two show views of the apparatus and interferometer from webcams on before and above the apparatus. The window on the upper left that is a serial interface to the Arduino based circuit similar to the previous one to unblock the lasers and apply power to the detectors. Three other windows are the serial interfaces to rotational mounts. On the right is a window with the Matlab platform that runs the data acquisition while scanning a piezoelectric element that changes the path-length difference in the interferometer.

2.1.1 Summary of Commands

The apparatus is basically the previous lab's apparatus plus

- **Pump Laser ON (Unblocked)**: On the command line of the Arduino platform type "1" . Turn it on before you start the experiments. You can do two visual verification that the beam is hitting the polarizer mount.

- **Pump Laser OFF (Blocked)**: On the command line of the Arduino platform type "2" . Turn it off once you are done with the lab session.

- **Detectors ON**: On the command line of the Arduino platform type "3" . Turn it on before you start the experiments. This voltage should be close to 5 V. The value
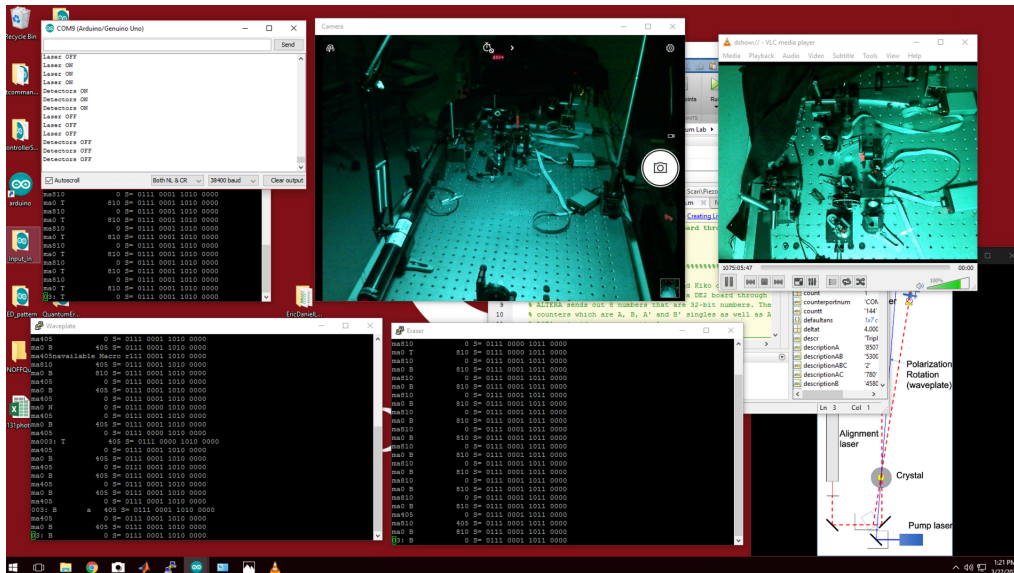
Figure 2: Screen of the Desktop that controls the experiment in the lab. It has a webcam window (upper-left), a serial interface window (lower left) and a Matlab window (right).

should be slightly above 5 V but never below 4.5 V (otherwise the detector signals will be unstable).

- **Detectors OFF**: On the command line of the Arduino platform type "4" . Turn it off once you are done with the lab session.

- **Alignment Laser ON (unblocked)**: On the command line of the Arduino platform type "5". **Before you do this you must be sure that the detectors are off**: the digital display seen with the webcam must read 0.000.

- **Alignment laser OFF (blocked)**: On the command line of the Arduino platform type "6".

- **Cold Restart**: In case you need to do this, in the command bar there is a round icon for the Arduino platform. You click on this. Then you click a right arrow in the upper left that compiles the program and loads it to the circuit. You then need to go to the top menu bar, click "Tools" and then click "Serial Monitor." This opens the screen that you see in the figure. Resize it and put it in the lower left. You can then minimize the Arduino platform.

- **Menu** On the command line of the Arduino platform type "0" for a display of the menu of commands.

- **Block Arm**: To block one of the arms of the interferometer we use the serial communications platform "Blocker" and enter "ma810" ($90° = 90 \times 9 = 810$ nods)

7

- **Unblock Arm**To unblock the arm enter "ma0" in the serial communications platform "Blocker."

- **Rotate waveplate**: To rotate the waveplate we use the serial communications platform "Waveplate" and enter "ma" followed by the angle in nods (rounded). For example $45° = 45 \times 9 = 405$ nods. So to rotate to 45 degrees you enter "ma405". Note that the waveplate rotates by twice the angle, so setting the waveplate to 45 degrees rotates the polarization by 90 degrees.

- **Insert Eraser**: To move the eraser we need to rotate the arm that holds it. To insert it in the path of the beam you need to rotate it 90 degrees, or enter enter "ma810" in the serial communications platform "Eraser". To take it out you enter "ma0".

3 Procedure

3.1 Planning

You need to plan ahead your experimental procedures. They should follow the following goals

1. If we block one of the arms of the interferometer, what is the predicted probability for reaching B and C? Investigate this experimentally. What does this tell you about the interferometer? Note that in the write-up we present the theory for an idealized experiment. The optical elements may not have the values that we assume to have (e.g., t may not be 0.707. All we know is that $|r|^2 + |t|^2 = 1$).

2. Equation 19 gives the number of counts as a function of the interference phase δ. Your goal is to take measurements and obtain values of V for various degrees of distinguishability. Compare those values with the prediction of Eq. 17.

3. When we add the eraser, the probability is given by Eq. 26. How close does the experiment reflect Eq. 26? Can you explain the difference between the counts in AB and AC?

3.2 Experiments

1. Using the command list given above, explore all the movable parts. Make sure that they do what you think they should do.

2. Take data scans according to your experimental plan.

3. In your report answer all questions and show all graphs with error bars and legends. Comment on their significance.

4. Return all windows and equipment to their initial settings: lasers blocked, detectors off, angles at 0.