# Lab 6 Polarization Entanglement - Remote Lab

## Phys434L Quantum Mechanics Lab
## 2021

May 29, 2021

## 1 Theory

In this lab we are going to study the ultimate quantum mechanical object: two-particle entanglement. We have already worked with entangled states and manipulated them in the delayed-choice experiment. Here we will bring the topic of entanglement in a more direct way. It is ironic that Einstein in trying to prove quantum mechanics wrong thought out this marvelous situation, which Schrödinger later called entanglement. Two particles are entangled when their wavefunctions cannot be factorized. However, when we make a measurement on one particle, it instantaneously defines the state of the other particle. At least, that is quantum mechanics says...

## 1.1 State Production

In this lab we will create photon pairs entangled in polarization. If we denote the polarization states of a photon, horizontal and vertical, as $|H\rangle$ and $|V\rangle$, respectively, then the photon pairs that we created in previous labs would be in the state

$$|\psi_i\rangle = |V\rangle_1|V\rangle_2. \tag{1}$$

This state is called a "product state" because the wavefunction of the pair is the product of the wavefunctions of the two particles.

The state of the two photons can be put in matrix form as

$$|\psi_i\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{2}$$

As we have seen before, a polarizer projects the state of the light into the direction of the transmission axis of the polarizer. If the polarizer is rotated an angle $\theta$ relative to the

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horizontal, then the polarizer eigenstates of transmission are

$$|\theta\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{3}$$

Thus, if we decide to measure the two photons with polarizers set to angles $\theta_1$ and $\theta_2$, as shown in Fig 1, then effectively the photons are projected into the state

$$|\psi_p\rangle = |\theta_1\rangle|\theta_2\rangle = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \otimes \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_1\cos\theta_2 \\ \cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 \\ \sin\theta_1\sin\theta_2 \end{pmatrix}. \tag{4}$$
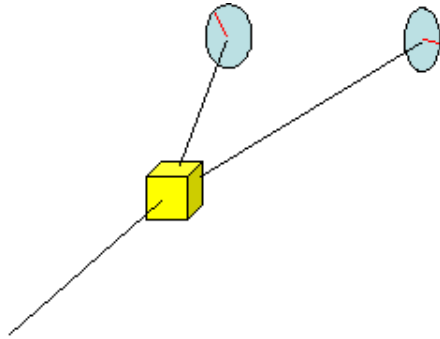


Figure 1: Down-conversion production of photons pairs and their detection with polarizers.

The probability of joint detection (i.e., of detecting both photons past the polarizers) is the square of the probability amplitude of being in state $|\phi_p\rangle$

$$P_p = |\langle\psi_p|\psi_i\rangle|^2. \tag{5}$$

The polarization state of the down-converted light that we created in previous labs was due to a pump beam that was horizontally polarized incident on an appropriately oriented crystal. By adjusting the crystal we can have down-converted pairs at 804 nm come out forming an angle with the incident direction, as shown in Fig 1. If we change the polarization of the pump to vertical we would not get down-conversion. However, if we rotate the crystal by 90° we get horizontally polarized pairs.

A few years ago Paul Kwiat (U. Illinois) came up with a clever trick: to put two thin down-conversion crystals back to back but rotated by 90° with respect to each other. He then sent a pump beam polarized at 45° to the pair of crystals, as shown in Fig. 2. This way the horizontal component of the pump polarization produces vertically polarized pairs with one crystal and the vertical component produces horizontally polarized pairs with the other crystal. If crystal thickness is smaller than the beam width, then there is no way to tell
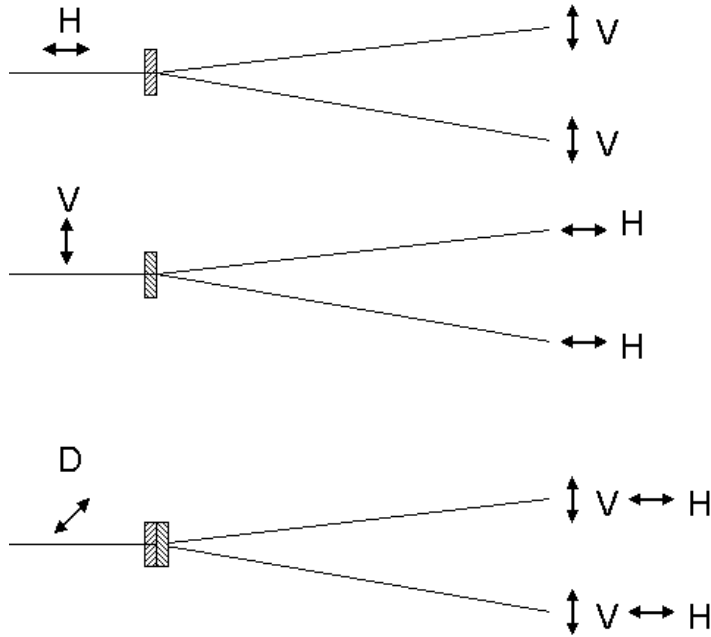
Figure 2: Method to produce polarization-entangled states: the bottom setup is a super-position of the two cases above.

in which crystal the photon pairs were created. Thus when the paths are indistinguishable the photon pairs get created into a state that is a superposition of the two possibilities:

$$|\Phi_{ent}\rangle = \frac{1}{\sqrt{2}} \left( |H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2 e^{i\delta} \right), \tag{6}$$

where $\delta$ is a phase between the two possibilities. For simplicity, and without much loss of generality let us assume that $\delta = 0$. The state is then

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} \left( |H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2 \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{7}$$

The skeptical physicist may say: "What about a mixed state?" A mixed state would be the situation where the light is not in a superposition of both horizontal and both vertical. Rather, half the time the pairs come horizontal and half the time they come vertical. How can we distinguish the two? The answer to this questions leads us straight to Bell.

The Dirac notation cannot describe a mixed state so we need to resort to another artifact of quantum theory: the density matrix. For a pure state $|\psi\rangle$ the density matrix is defined as

$$\hat{\rho}_\psi = |\psi\rangle\langle\psi|. \tag{8}$$

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That is, it is the outer product of the state vector with itself. As an example consider the state $|\Phi^+\rangle$. It is given in matrix form by

$$\hat{\rho}_{\Phi^+} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

Before we discuss Bell let us study in more detail the entangled state given by Eq. 7. What would the form of the state be in the diagonal-antidiagonal basis ($|+45\rangle \equiv |D\rangle$ and $|-45\rangle \equiv |A\rangle$)? If you recall, the diagonal basis states are related to the horizontal-vertical states by:

$$|D\rangle = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle) \tag{10}$$

$$|A\rangle = \frac{1}{\sqrt{2}} (-|H\rangle + |V\rangle). \tag{11}$$

If we put $|H\rangle$ and $|V\rangle$ in terms of $|D\rangle$ and $|A\rangle$ for each particle in Eq. 7, we can show that

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|D\rangle_1 |D\rangle_2 + |A\rangle_1 |A\rangle_2). \tag{12}$$

In the H-V basis the photons are in a superposition of being parallel to each other in two different ways. In the rotated basis they are also in an entangled state that is a superposition of the two possibilities in which they are parallel! This is an interesting but unique aspect of state $|\Phi^+\rangle$. Let us verify this the following way

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|D\rangle_1 |D\rangle_2 + |A\rangle_1 |A\rangle_2) \tag{13}$$

$$= \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{14}$$

State $|\Phi^-\rangle$ corresponds to state $|\Phi_{ent}\rangle$ of Eq. 6 with $\delta = \pi$. In the diagonal basis $|\Phi^-\rangle$ becomes

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}} (|D\rangle_1 |A\rangle_2 + |A\rangle_1 |D\rangle_2). \tag{15}$$

That is, in state $|\Phi^-\rangle$ the light switches from being parallel in the H-V basis to being orthogonal in the D-A basis.

The density matrix for the state is a bit ominous if you apply Eq. 8 to the state of Eq. 4. The probability that a photon in a state $\phi$ is measured to be in state $\psi$ (the absolute-value squared of the inner product) is given by the trace of the product of the two density matrices. The trace is the sum of the diagonal elements of the product matrix:

$$P(\phi)_\psi = |\langle \psi | \phi \rangle|^2 = \mathrm{Tr}(\hat{\rho}_\psi \hat{\rho}_\phi) \tag{16}$$

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The above probabilities are the ones predicted by entanglement. You can see that the detection of one is correlated with the result of the detection of the other one. It is laborious, but can be shown that

$$P(\theta_1, \theta_2)_{\psi_p} = \frac{1}{2} \cos^2(\theta_1 - \theta_2) \tag{17}$$

In inspecting Eq. 17 you can see that we get maximum probability when the two angles are the same. That is, the photon pairs in state $|\Phi^+\rangle$ are parallel in *any* basis. As soon as we measure one of the photons to be polarized along one direction (with probability 1/2) we get that the other one is polarized in the same direction with unity probability. Thus, one can think of the "1/2" in Eq. 17 as the probability of detecting the first photon, and the cosine term as the conditional probability of detecting the other given that the first one was detected at the angle $\theta_1$.

One last point. The "1/2" term stems from the randomness of quantum mechanics. When we detect the polarization of the first photon anything can happen. It may be transmitted or not. It is the detection of the second one that is conditional to the *result* of the first detection. Thus, this is not faster-than-light communication of *information*, because we do not control the outcome of the first measurement.

In the lab we measure the *coincidences*, which is equivalent to the *conditional* probability of finding photon 1 in state $|\theta_1\rangle$ given that photon 2 was found in state $|\theta_2\rangle$.

$$P(\theta_1/\theta_2)_{\psi_p} = \frac{P(\theta_1, \theta_2)_{\psi_p}}{P(\theta_2)_{\psi_p}} \tag{18}$$

The conditional Probability takes care of the 1/2:

$$P(\theta_1/\theta_2)_{\psi_p} = \cos^2(\theta_1 - \theta_2) \tag{19}$$

This correlation is the basis for *nonlocality*; that the detection of one photon immediately "collapses" the wavefunction of the two, instantaneously at faster than the speed of light. This is the view advocated by Bohr in the so called "Copenhagen interpretation" of quantum mechanics, which Einstein criticized and derided as "spooky action at a distance."

How do we know that we are not in a mixed state? This is the "realistic view." In this view the photons had their state of polarization defined before the measurement was done. The mixed state is the probabilistic combination of two or more pure states. For the case of the mixture of state $|\psi\rangle$ with probability $\mathcal{P}_\psi$ and state $|\phi\rangle$ with probability $\mathcal{P}_\phi$, with $\mathcal{P}_\psi + \mathcal{P}_\phi = 1$,

$$\hat{\rho}_m = \mathcal{P}_\psi |\psi\rangle\langle\psi| + \mathcal{P}_\phi |\phi\rangle\langle\phi| \tag{20}$$

As an example, let us find the mixed state mentioned earlier: the realistic alternative

to the $|\Phi^+\rangle$. It entails the product states each with a probability of $1/2$

$$\hat{\rho}_{HH,VV} = \frac{1}{2}|H\rangle_1|H\rangle_2\langle H|_1\langle H|_2 + \frac{1}{2}|V\rangle_1|V\rangle_2\langle V|_1\langle V|_2 \tag{21}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{22}$$

Note that this matrix is different than the one of Eq. 9, which means that we can use it to tell the difference between the two situations.

The conditional probability of detection predicted for the mixed state is given by

$$P(\theta_1/\theta_2)_{\text{mix}} = \cos^2\theta_1\cos^2\theta_2 + \sin^2\theta_1\sin^2\theta_2. \tag{23}$$

Notice that Eqs. 17 and 23 have a different functional form. Thus, we have a chance to find out which one is correct. When $\theta_2 = 0$ (i.e., finding photon 2 in state $|H\rangle_2$) both give the same answer:

$$P(\theta_1/H_2)_\psi = P(\theta_1/H_2)_{\text{mix}} = (1/2)\cos^2\theta_1. \tag{24}$$

However, if $\theta_2 = \pi/4$ (i.e., finding photon 2 in state $|D\rangle_2$) they give a different answer:

$$P(\theta_1/D_2)_\psi = \cos^2(\theta_1 - \pi/4), \tag{25}$$

while

$$P(\theta_1/D_2)_{\text{mix}} = 1/2. \tag{26}$$

In this lab we will do this measurement.

One final comment. There are two other states that also follow similar results. These are described by the state

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2\right). \tag{27}$$

The four set of states $|\Phi^\pm\rangle$ and $|\Psi^\pm\rangle$ are also known as the "Bell" states. They have interesting properties too: photons in state $|\Psi^-\rangle$ are in a superposition of possible ways of being orthogonal in *any* basis, while photons in state $|\Psi^+\rangle$ are orthogonal in the H-V basis and parallel in the D-A basis.

## 1.2 Bell Inequalities

This argument started by Einstein, Podolski and Rosen, and the reply by Bohr and Schödinger generated much debate since it was first presented, 1935. Yet for most scientists it was considered mostly an academic hypothetical argument with no possibility of experimental verification. The correlations described earlier are a "look ahead" into

today's understanding, but the debate raged on for years. That changed dramatically in 1964, when John Bell devised a rigorous proof regarding situation where these two distinct philosophical possibilities could given a conclusive answer by an experiment. This "woke up" the community and researchers started setting out to do the experimenters to resolve the dilemma. Today, after a long series of landmark experiments, the issue is all settled: quantum mechanics is correct. Nature is not realistic, and also "nonlocal" (the instantaneous snapping of the state of the other photon, or "spooky action at a distance"). We will do this experiment here, and see it for ourselves that nature is not-realistic and nonlocal.

Complement 8.B of the textbook presents a discussion of one Bell-type test known as the "Clauser-Horne" Bell-inequality test. Here we will use a different one known as the Clauser-Horne-Shimony-Holt (CHSH) inequality, after the names of the authors. We begin by defining a variable $E$ that expresses the correlation between the polarizations of the two photons. If they are perfectly correlated, as is the case in state $|\psi\rangle$, then $E = 1$. If they are uncorrelated then $E = 0$. The expectation value of the correlation can be defined for arbitrary angles $\theta_1$ and $\theta_2$. At these angles $E$ is given by

$$E(\theta_1, \theta_2) = P(\theta_1, \theta_2) + P(\theta_1 + \frac{\pi}{2}, \theta_2 + \frac{\pi}{2}) - P(\theta_1, \theta_2 + \frac{\pi}{2}) - P(\theta_1 + \frac{\pi}{2}, \theta_2), \quad (28)$$

For state $|\psi\rangle$ it can be shown that the expression for $E$ reduces to

$$E_\psi(\theta_1, \theta_2) = \cos[2(\theta_1 - \theta_2)] \quad (29)$$

The perfect correlation of the state $|\psi\rangle$ is manifested in Eq. 29 because $E(\theta, \theta) = 1$ regardless of $\theta$. In contrast, the mixed state gives a different correlation:

$$E_{\text{mix}} = \cos(2\theta_1)\cos(2\theta_2). \quad (30)$$

Therefore, one can find situations where the two expectation values give different results.

**Question 1** Calculate the correlation parameters $E(0,0)$ and $E(\pi/4, \pi/4)$ for both the entangled state and the mixed state. State your conclusions about the results.

Note that to obtain $E$ we need to do measurements pairwise at the combinations of the 4 angles: $\theta_1$, $\theta_2$, $\theta_1 + \pi/2$ and $\theta_2 + \pi/2$. Furthermore, CHSH defined a variable $S$ that depends on 4 angles, two for each photon. It is given by

$$S = E(\theta_1, \theta_2) - E(\theta_1, \theta_2') + E(\theta_1', \theta_2) + E(\theta_1', \theta_2'), \quad (31)$$

where $\theta_1$ and $\theta_1'$ are two polarizer angles for photon 1, and similarly, $\theta_2$ and $\theta_2'$ are two angles for photon 2. Given that the measurements of $E$ require the orthogonal angles too, then to get a value for $S$ we need to make 16 pairwise measurements. A local realistic theory *satisfies*

$$|S| \leq 2. \quad (32)$$

Entangled states measured at certain angles will violate this inequality.

**Question 2** Show that the inequality of Eq. 32 is violated for state $|\psi\rangle$ when $\theta_1 = -\pi/4$, $\theta_1' = 0$, $\theta_2 = -\pi/8$ and $\theta_2' = \pi/8$.

**Question 3** Show that the inequality of Eq. 32 with $E_{mix}$ of Eq. 30 is satisfied when $\theta_1 = -\pi/4$, $\theta_1' = 0$, $\theta_2 = -\pi/8$ and $\theta_2' = \pi/8$.

The measurement of the parameter $S$ that provides the maximal violation of the Bell inequality of Eq. 32, can be done for state of Eq. 7 with the angles: $\theta_1 = 0°$, $\theta_1' = 45°$, $\theta_2 = 22.5°$, $\theta_2' = 67.5°$. Because of a lack of 100% detector efficiency the correlation parameters have to be normalized as follows:

$$E(\theta_1, \theta_2) = \frac{N(\theta_1, \theta_2) + N(\theta_1 + \frac{\pi}{2}, \theta_2 + \frac{\pi}{2}) - N(\theta_1, \theta_2 + \frac{\pi}{2}) - N(\theta_1 + \frac{\pi}{2}, \theta_2)}{N(\theta_1, \theta_2) + N(\theta_1 + \frac{\pi}{2}, \theta_2 + \frac{\pi}{2}) + N(\theta_1, \theta_2 + \frac{\pi}{2}) + N(\theta_1 + \frac{\pi}{2}, \theta_2)}, \quad (33)$$

where $N(\theta_1, \theta_2)$ is the number of coincidences in a set amount of time when the polarizer angles are $\theta_1$ and $\theta_2$.

An important quantity is the uncertainty in $S$, given by

$$\Delta S = \sqrt{\sum_{i=1}^{16} \left( \Delta N_i \frac{\partial S}{\partial N_i} \right)^2}, \quad (34)$$

where the uncertainties in the counts are due to statistical errors: $\Delta N_i = \sqrt{N_i}$.

## 2 Parts

The parts that are needed are listed in the table below and refer to Fig. 4

| Qty | Part | Description/Comments |
|---|---|---|
| 1 | BBO crystal pair | (X) Two type-I BBO crystals 0.5 m thick, with 29° phase matching rotated 90° relative to each other and mounted on a rotation plus tilting (mirror-like) mount. Axes of the crystals must be in horizontal/vertical planes. |
| 1 | Half-wave plate | (V) 405 nm, zero order. |
| 1 | Compensating crystal | (U) Quartz, A-cut, 6-8 mm thick on a motorized rotation mount. |
| 1 | Quartz plate | Inserted in/out in the path of one of the photons to produce a mixed state. Motorized. |
| 2 | Half-wave plate | (H) 810 nm, zero order, on motorized stage. |
| 2 | Prism polarizer | (P) Glan-Thompson. |

# 3 The Remote Apparatus

## 3.1 General

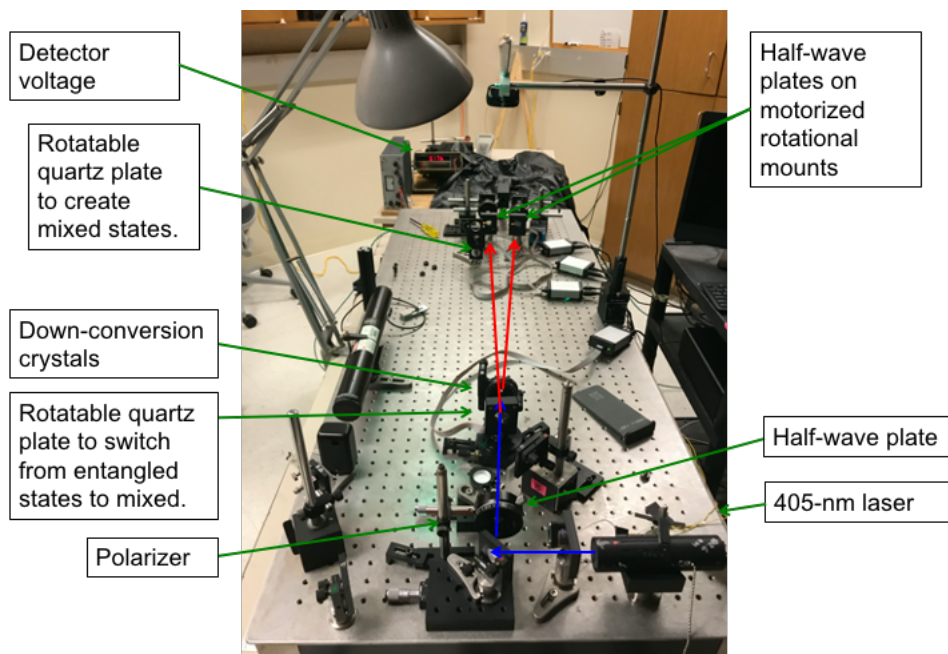The apparatus is shown in Fig. 3. As can be seen, the layout is simpler than the ones for



Figure 3: View of the apparatus and all of its components. The path of the 405-nm laser beam is shown in blue, and the path of the down-converted photons is shown in red. The picture is missing the second quartz crystal (a late addition).

interference experiments. The schematic of Fig. 4 shows more details.
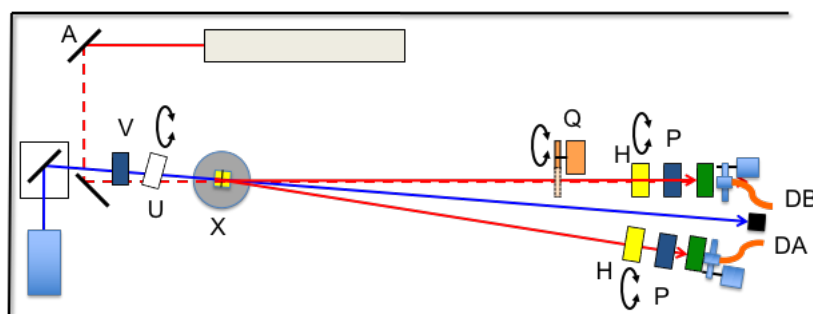


Figure 4: Apparatus for measuring entanglement and Bell Inequalities.

In the experiment a 405-nm laser goes through half-wave plate (V) that out the input light in state $|D\rangle$. A quartz crystal (U) is used to make the two possible states produced by the crystals (X), $|HH\rangle$ and $|VV\rangle$, indistinguishable, and for setting $\delta = 0$ in Eq. 6.

Each of the down-converted photons then is projected onto a linear polarization state. In the apparatus shown in Fig. 4, polarizers are preceded by half-wave plates. The reason for this is practical. The polarizers are bulky and wedge the beam, so if we rotate them, they modify the alignment of the light heading into the optical fiber. The waveplates, due to their polarization rotating properties, must turn by half of the angle that the rotating polarizer is expected to rotate. So in actuality the down-converted photons go through rotating half-wave plates (H) and fixed horizontal polarizers (P). Thereafter the photons go through filters, enter optical fibers and end up at the detectors. We refer to the detection of the state of the two photons by Alice and Bob. If we rotate by 90 degrees the quartz crystal (U), which is before the BBO, and insert an additional quartz crystal in the path of one of the photons after the BBO (Q), we render the photons in a mixed state.

## 3.2   Remote Hardware

In this experiment there are 5 actuator parts, with 4 being rotating:

- We have two quartz plates that in one configuration make the two possible polarization states ($|HH\rangle$, $|VV\rangle$) indistinguishable. They can be rotated 90 degrees to put the photons in a mixed state. They are controlled by two serial screens labeled "Crystal - COM12" and "EPR com8." To make the mixed states you have to enter "ma810" on both. To preserve the entangled state, you enter "ma0".

- Two rotating half-wave plates, one for each photon allow us to project the light into any linearly polarized state. They are "Alice - COM4" and "Bob - COM15." Their rotation is specified by the angle in units of 1/9 of a degree (inventing a unit, we call it "nods"). This way 90 nods = 10 degrees.

- This last actuator is electrical, and like in previous labs, it is used to turn on/off the laser and detectors. It uses Arduino driving two relays. Just in case, the program name is "ONOFF2021."

### 3.2.1   Logging in

You log into the lab computer using remote desktop. It is the same PC machine as the first 3 labs:

Ho313-96143W.colgate.edu

Then you log into the PC:
Username: phas2
Password: Fr2=Gm1m2

### 3.2.2 Actuators

Once you log in, your screen will appear as something like what you see in Fig. 5. You will see 7windows. One is a view of the apparatus from one of two possible webcams (upper center in the photo). The window on the upepr left is a serial interface to the Arduino based circuit tto turn ON/OFF the power to the pump laser and detectors, as in previous labs. More on this below. In the window with the Matlab platform there are 2 programs we will use: one to acquire data continuously (Continuous Counting), and the second program (Bell_Manual_Altera) to do a paused scan of a specified number of points (similar to what you did in a previous lab). There are 4 other serial windows that control the rotation of elements (below). They work the same way as in previous labs.
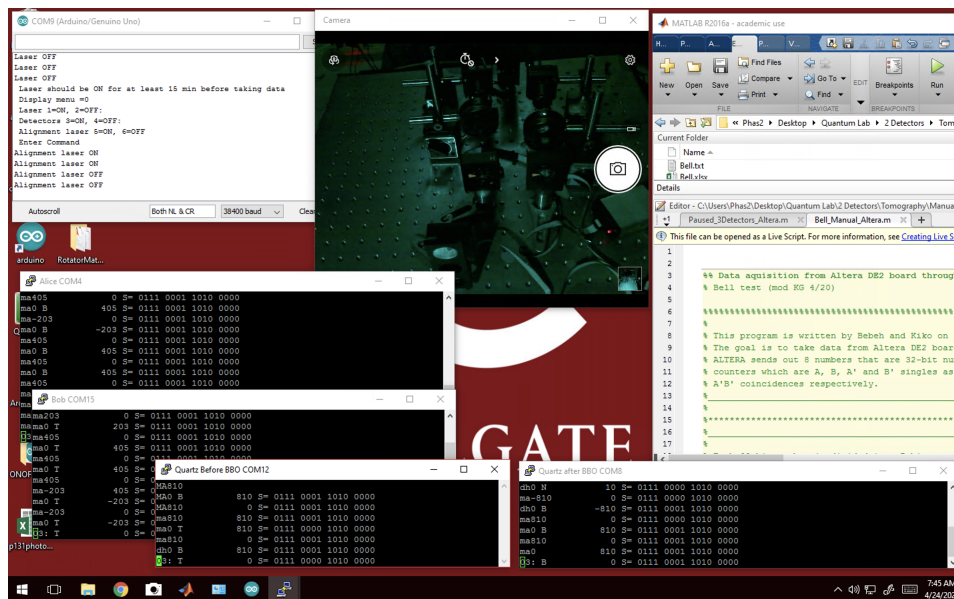


Figure 5: Screen of the Desktop that controls the experiment in the lab. The computer name is: "Ho313-96143W.colgate.edu". It has a webcam window (upper-left), 4 serial interface windows (lower portion) and a Matlab window (upper right).

### 3.2.3 Webcam

The webcam allows you to see the parts of the apparatus. Note that there is visual evidence that the laser is ON (by the glare in the beam dump) and that detectors are ON by the voltage applied to them. We have *two webcams*. One gives an overall view of the apparatus so you can see the beam's glare and the detector voltage, and the other view is a close-up of the waveplates, so you can see them move as you enter commends to rotate them. You can switch cameras by clicking on an icon on the upper left of the webcam window.

## 3.3 Actuator Commands

Here we detail the commands for the different programs. All commands given below are within quotation marks. **Do not enter the quotation marks**.

### 3.3.1 Laser and detectors ON/OFF

The laser is blocked for safety reasons. This feature allows the laser to be unblocked during the experiment. The power to the laser and detectors is controlled by a program that runs on an Arduino-based circuit (through the COM9 serial port). The 2 detectors (Alice-right and Bob-left) can be turned ON or OFF at any time. You can see the detector voltage in the digital display seen in the webcam view. You should turn them OFF when you are done.

The Arduino program is always running. The command is entered on the top command line. The action taken is listed in the lower part of the window. A summary of commands follows:

- **Menu**: Type "0" for a menu of commands.

- **Laser ON**: On the command line type "1" . Turn it on before you start the experiments. You can do two a visual verification via the webcam that this action has taken place: you can see a purple glare coming out of the beam dump.

- **Laser OFF**: On the command line type "2" . Turn it off once you are done with the lab session.

- **Detectors ON**: On the command line type "3" . Turn it on before you start the experiments. There is a red digital readout near the top-central part of the webcam window that indicates how much voltage is being applied to the detectors. This voltage should be close to 5 V. The value should be slightly above 5 V but never below 4.5 V (otherwise the detector signals will be unstable).

- **Detectors OFF**: On the command line type "4" . Turn it off once you are done with the lab session.

- **Alignment Laser ON**: Type "5" to turn this laser on (Unblock). You can see its glare on the optical pieces. It is not really needed for this lab. It must be off before you turn the detectors on.

- **Alignment Laser OFF**: Type "6" to block the laser.

- **Cold Restart**: In case you need to do this, in the command bar there is a round icon for the Arduino platform. You click on this. You must load the program "ONOFF2021" if not already loaded.Then you click a right arrow in the upper left that compiles the program and loads it to the circuit. You then need to go to the top menu bar, click "Tools" and then click "Serial Monitor." This opens the screen that

you see in the figure. Resize it and put it in the lower left. You can then minimize the Arduino platform.

### 3.3.2  Rotational Controls

We have 4 rotational mounts controlled by serial interfaces (the ones with a black background). The way to move the motor is to enter "ma" followed by the angle in degrees times 9 (i.e. in "nods," so 10 degrees is 90 nods, and 90 degrees is 810 nods). Once you enter the command, a line-record of the instruction appears. This is also useful to keep track of what was the previous instruction. It is important to verify that the rotator is moving. There is a small mark on the top side of rim of the rotator that rotates with the optical element.

## 3.4  Data Scans

This is the standard Matlab program to do scans. The directory is: It has two programs:
"ContinuousCounting_Altera" in directory

```
\Desktop\Quantum Lab\2 Detectors\Continuous Counting\No Piezo\
```

and
"Bell_Manual_Altera" in directory

```
\Desktop\Quantum Lab\2 Detectors\Tomography\Manual Waveplate\
```

The first program basically is a free-running program. You only need to enter the COM port (COM6). The second program does a stepped scan for a specified number of points and for a specified time per point. For example for a Bell measurement you need 16 points (it helps to enter a couple of more, in case you want to retake a point. It pauses in between points so that you can change the waveplate angles in between points. At the start a menu will appear with several entries:

- **COM Port of Counter**: Must read read "COM6"

- **Number of intervals**: Enter the number you need (e.g. 18). Maximum is 20.

- **Time per interval**: Enter the number of seconds per point (e.g. 1 for something quick, but more for better statistics).

- **Name of File**: Enter file name. (The data will be placed in a spreadsheet file with the name you enter.)

- **OK button**: The program will take a data point after you click this, and then pause.

# 4  Procedure

The procedure involves the following steps: The apparatus is set up so that the photon pairs are in the entangled state of Eq. 7.

1. **First measurements**: Run the program "ContinuousCounting_Altera," available in one of the Matlab tabs. Enter COM6 for the Counter COM port. Arrange the screens so you can see simultaneously the webcam, the continuous counting GUI and the two serial ports for Alice and Bob, as shown in Fig. 6. The objective it to take measurements in enough settings of the waveplates so that you feel if the photons are in an entangled state that satisfies Eqs. 7 and 12. The waveplates are in front of fixed polarizers, but act as a rotating polarizer. The only difference is that the angle you specify for the waveplate is equivalent to a polarizer set at twice the angle (e.g., setting the waveplate at 45 degrees is equivalent to rotating the polarizer by 90 degrees).

   **Question 4** Show your results and state your conclusions.

   .

2. **Angle correlations**: Now we want to be more systematic.The correlations for an entangled state predict the conditional probabilities of Eq. 19, 24 and 25; whereas the mixed state predicts the probabilities of Eqs. 23, 24 and **??**. Take a manual scan were you vary some waveplate angle over a range of values such that you can sort out which state we have: entangled or mixed. To do this:

   (a) Run "Bell_Manual_Altera."

   (b) The number of angles in your scan (maximum is 20).

   (c) Enter the the COM port (COM6) and the time per second.

   (d) Position the screens so that you can change the rotator positions while you observe the rotator in the webcam move and data plot. See Fig. 6.

   (e) Set the waveplate you will change in the appropriate angle in the appropriate serial window in nods (degrees times 9). Remember that the waveplate rotates the polarization by twice the angle. You may want to make a table of waveplate angles in nods beforehand.

   (f) After clicking "OK" in the window "Waveplate Orientation" the program will record a data point. You do not need to enter anything in the dialog window. Just click "OK". See the data point appear on the screen.

   (g) Set the next angle and repeat e-g until all points are taken. If you abort, Matlab does not like it. You may have to restart Matlab.

3. **Bell test** Finally the big test on the nature of reality. Here you follow instructions.
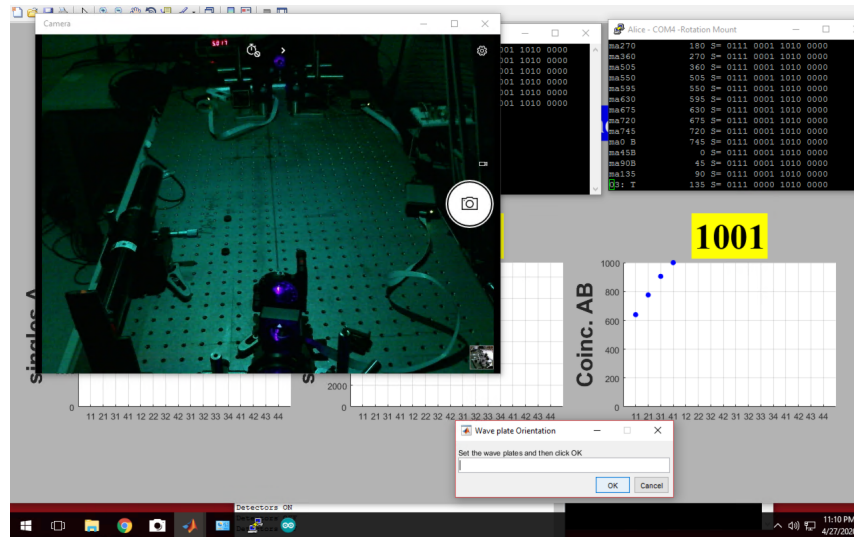
Figure 6: Example of Desktop screen for taking data step by step.

(a) Run again "Bell_Manual_Altera."

(b) For better statistics, enter a much longer integration time per point (much more than 1 s; there are only 16 points needed, but you can enter a couple more, just in case you want to repeat one or two points.)

(c) Take data and record the results for the angles of Bob and Alice given in the table below.

(d) Follow the same data acquisition steps as in the previous part.

| $\theta_{\text{Bob}}$ (degrees) | $\theta_{\text{Alice}}$ (degrees) | $\text{HWP}_{\text{Bob}}$ (nods) | $\text{HWP}_{\text{Alice}}$ (nods) | $N(\theta_{\text{Bob}}, \theta_{\text{Alice}})$ |
|---|---|---|---|---|
| 0 | 22.5 | 0 | 101 | |
| 45 | 22.5 | 203 | 101 | |
| 90 | 22.5 | 405 | 101 | |
| 135 | 22.5 | 608 | 101 | |
| 0 | 67.5 | 0 | 304 | |
| 45 | 67.5 | 203 | 304 | |
| 90 | 67.5 | 405 | 304 | |
| 135 | 67.5 | 608 | 304 | |
| 0 | 112.5 | 0 | 506 | |
| 45 | 112.5 | 203 | 506 | |
| 90 | 112.5 | 405 | 506 | |
| 135 | 112.5 | 608 | 506 | |
| 0 | 157.5 | 0 | 709 | |
| 45 | 157.5 | 203 | 709 | |
| 90 | 157.5 | 405 | 709 | |
| 135 | 157.5 | 608 | 709 | |

4. **Analysis**

   (a) Calculate $S$ from the data in the table above. You can do so in Mathematica or by entering the values in the spreadsheet that is provided (for this you enter the data in the tab labeled "CheatsheetHWP", to then get the value of S in the HH+VV tab).

   (b) Calculate the error in your determination of $S$. (Done also in the spreadsheet.) State your conclusions.

   (c) Rotate the 2 quartz plates (U and Q) by 90 degrees by entering "ma810" in each of the serial windows. Verify that the plate rotates in the webcam. This should put the light into a mixed state.

   (d) Take data that makes a convincing case that the photons are no longer entangled.

   (e) Retake the data and evaluate $S$ for this situation.

   (f) Write a conclusion based on your results.