Lab 2 Photon Stern-Gerlach's - Week 4

Phys434L Quantum Mechanics Lab
2018

February 18, 2018

1 Introduction

Photons' polarization is a 2-dimensional space, similar to the space of spin-1/2 particles. In this lab we will mimic Stren-Gerlach experiments with spins with the polarization of photons. We will treat the states of the photons using the vector algebra of quantum mechanics. The basis of vectors is then represented by two-element column vectors. The simplest basis is the one where we consider the polarization of the light aligned with the horizontal and the polarization vertical axes. Following the notation of the textbook,

$$|\mathrm{H}\rangle \xrightarrow[HV \text{ basis}]{} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1}$$

for the state of a horizontally polarized photon, and

$$|\mathrm{V}\rangle \xrightarrow[HV \text{ basis}]{} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

for the state of the vertically polarized photon. Down-converted photons are produced in state $|V\rangle$.

Optical elements will act as operators, and so will be expressed as 2×2 matrices. Some of the polarization elements have local axes H', V', so let us define a local set of axes that is rotated by an angle θ relative to the H, V basis. This way then

$$|\mathrm{H}'\rangle \xrightarrow[HV \text{ basis}]{} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{3}$$

represents the state of a horizontally-polarized photon in the local basis of the optical element, and

$$|\mathrm{V}'\rangle \xrightarrow[HV \text{ basis}]{} \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \tag{4}$$

for the vertically-polarized component.

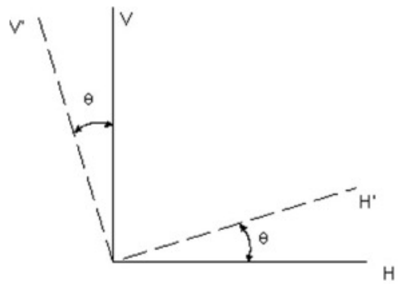

Figure 1: Local axes of rotated components.

1.0.1 Equipment

In addition to the setup from the previous lab the new parts needed are:

Qty	Part	Description/Comments
1	Polarizing beam splitter	Replace previous beam splitter.
1	Half-wave plate	Mounted and calibrated.
1	Polarizer	Glan-Thompson type. Mounted and calibrated.
1	Quarter-wave plate	Mounted and calibrated.

Note: All positive angles are measured relative to the horizontal, for counter-clockwise rotations when looking into the beam.

2 Experiment 1: Transforming the state.

In this section we will investigate the effect of the half-wave-plate on photon states. Let us introduce our first optical element: the half wave plate. In the local basis, the expression for this operator is

$$\hat{W} \xrightarrow[H'V' \text{ basis}]{} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5}$$

This optical device inserts a phase of π between the horizontal and vertical components of the light in the local axes. Since we are working in the H, V basis we need to transform the operator to that basis. It can be shown that the transformation matrix is:

$$\hat{T} = \begin{pmatrix} \langle H|H'\rangle & \langle H|V'\rangle \\ \langle V|H'\rangle & \langle V|V'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{6}$$

Question 1 Take home: Show that the matrix for the half-wave plate in the H, V basis is given by:

$$\hat{W}(\theta) \xrightarrow[HV \text{ basis}]{} \hat{T}\hat{W}\hat{T}^{\dagger} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \tag{7}$$

You can do this with Mathematica, but then you have to show your code. Individual work here!

1. The apparatus is the same as the previous lab. Put the half-wave plate forming an angle of 22.5° in the path of the light.

2. We need to make a change in the setup: replace the beam splitter by a *polarizing* beam splitter. This beam splitter transmits horizontally polarized light and reflects vertically polarized light. Align the beam splitter and collimator C so that you get counts on the C detector. The apparatus should look like the one shown in Fig. 2.

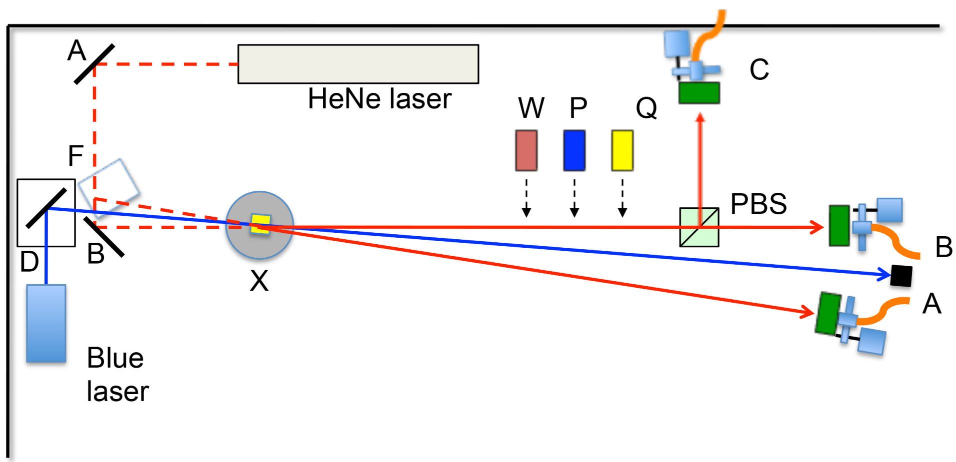

Figure 2: Using a Polarizing beam splitter (PBS) to deflect photon polarization by path. State transforming elements are half-wave plate (H), polarizer (P) and quarter-wave plate (Q).

The role of the polarizing beam splitter followed by the detector is to do measurements on the H, V basis. If the state input to the beam splitter is $|\psi\rangle$ then the transmitted and reflected light is: $N_H = N|\langle H|\psi\rangle|^2$ and $N_V = N|\langle V|\psi\rangle|^2$, respectively.

Question 2 In lab: show that $\hat{W}(0)|V\rangle = |V\rangle$ and $\hat{W}(45°)|V\rangle = |H\rangle$ to within an overall phase.

3. Record the average number of counts when $\theta = 0$ and $\theta = 45°$. Call these $N_{V\max}$ and $N_{H\max}$. We will use these to normalize the counts in other sections of the lab.

4. Take data for the following settings of the wave plate: from 0 to 180 degrees in increments of 15 degrees.

Question 3 Take home: Graph N_V vs. θ, plotting also the error bars that go with each point ($\sqrt{N}$), and plot it against the calculated probability$N_{V\max}|\langle V|\psi\rangle|^2$. Do a second similar plot for the data for N_H and the calculated $N_{H\max}|\langle H|\psi\rangle|^2$.

3 Experiment 2: Projection

In this case we are going to investigate the effect of a polarizer on photon states. A polarizer is represented by the following operator:

$$\hat{P} \xrightarrow[H'V' \text{ basis}]{} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{8}$$

Question 4 Take home: Find the matrix for $\hat{P}(\theta) \xrightarrow[HV \text{ basis}]{} \hat{T}\hat{P}\hat{T}^{\dagger}$.

Question 5 Take home: Show that $\hat{P}(\theta) \xrightarrow[HV \text{ basis}]{} = |H'\rangle\langle H'|$.

1. Replace the wave plate by a polarizer. Make sure that the polarizer is normal to the input beam. It should not "wedge" the beam.

2. Take data for settings of the polarizer set to angles 0 to 90°.

   Question 6 Take home: Calculate the probability for detection at the two outputs of the beam splitter.

3. Make a graph of the normalized detections on each detector (i.e., $H_V/N_{V\max}$ and $H_H/N_{H\max}$) in the same graph along with theoretical predictions.

   Question 7 Take home: Explain the previous graph. What does the polarizer do?

4 Experiment 3: Circular basis

Other important "qubit" states are the circular states: right circular,

$$|\mathrm{R}\rangle \xrightarrow[HV \text{ basis}]{} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}, \tag{9}$$

and left circular

$$|\mathrm{L}\rangle \xrightarrow[HV \text{ basis}]{} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{10}$$

Now consider a quarter-wave plate. This is another birefringent device, with matrix given by

$$\hat{Q} \xrightarrow[H'V' \text{ basis}]{} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \tag{11}$$

It introduces a phase of 90° between the two components of the local basis.

We can rotate the quarter-wave plate. The operator for the quarter-wave plate when it is rotated by 45° is

$$\hat{Q}(45^\circ) \xrightarrow[HV \text{ basis}]{} \frac{1}{2}\begin{pmatrix} 1+\mathrm{i} & 1-\mathrm{i} \\ 1-\mathrm{i} & 1+\mathrm{i} \end{pmatrix} \tag{12}$$

Suppose the situation of a photon with state $|\psi\rangle$ that goes through a quarter-wave plate and gets transmitted through the beam splitter. The probability is

$$\mathcal{P} = |\langle H|\hat{Q}(45^\circ)|\psi\rangle|^2. \tag{13}$$

It could be rewritten as

$$\mathcal{P} = |\langle\phi||\psi\rangle|^2, \tag{14}$$

where

$$|\phi\rangle = \hat{Q}(45)^\dagger|H\rangle = e^{-\mathrm{i}\pi/4}|L\rangle. \tag{15}$$

Similarly, for the reflected photon

$$\hat{Q}(45)^\dagger|V\rangle = e^{+\mathrm{i}\pi/4}|R\rangle. \tag{16}$$

The significance of this is that the quarter-wave plate in series with the beam splitter becomes a detector in the R, L basis.

1. Place the quarter wave plate oriented by 45 degrees before the beam splitter.
2. Place the half-wave plate before the quarter wave plate.
3. Use the arrangement to verify that the above conclusions are true. You need to take measurements. Suggestion: What if the half-wave plate is set to $\theta = 15^\circ$?

Question 8 Do you think that a g_2 test would give a value less than one here?