

Lab 3c Single-Photon Interference - Week 6

Phys434L Quantum Mechanics Lab
2018

March 13, 2018

1 Theory

We use a Mach-Zehnder interferometer. The interferometer has two 50-50 beam splitters and two mirrors. Consider the interferometer shown in the Fig. 1. Suppose the a photon

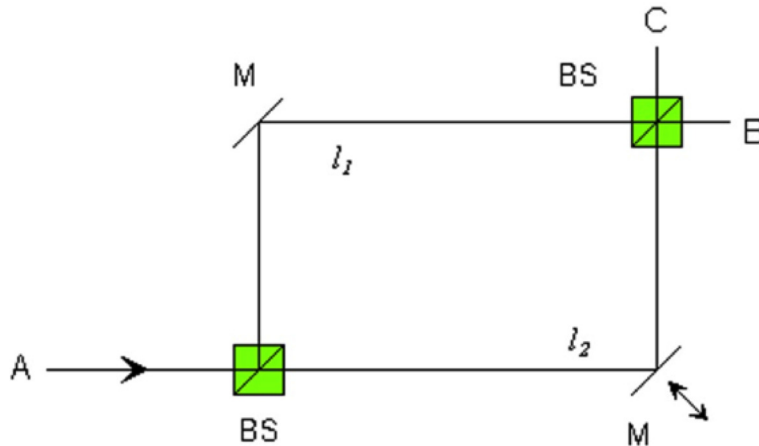


Figure 1: A Mach-Zehnder interferometer.

with a normalized wave-function $|\phi_0\rangle = |\phi_x\rangle$ is incident on the interferometer from A traveling along the x -direction. Eventually the photon will exit through either port B or port C. Thus, the final wavefunction past the second beam splitter will be

$$|\phi'_0\rangle = b|\phi_x\rangle + c|\phi_y\rangle, \quad (1)$$

where $|\phi_x\rangle$ and $|\phi_y\rangle$ are the normalized wavefunctions for the photon going in the directions x and y , respectively. The complex coefficients b and c are the probability amplitudes for the photon going through ports B and C, respectively. Normalization thus implies that

$$b^*b + c^*c = 1. \quad (2)$$

To construct expressions for b and c we need to consider how the photon gets to B and C . Since the optical components of the interferometer affect the light as it goes through them, then quantum mechanically we will represent them by operators. The beam-splitters either reflect or transmit a photon. The operators for a beam-splitter applied to the state vectors are

$$\hat{B}|\phi_x\rangle = r|\phi_y\rangle + t|\phi_x\rangle \quad (3)$$

$$\hat{B}|\phi_y\rangle = t|\phi_y\rangle + r|\phi_x\rangle. \quad (4)$$

where $r = i/\sqrt{2}$ and $t = 1/\sqrt{2}$. Notice that the probabilities for being reflected or transmitted are $1/2$, and that reflection carries a phase of $\pi/2$.

Question 1 In Lab: If the matrix representation of the state vectors is

$$|\phi_x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (5)$$

and

$$|\phi_y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (6)$$

what is the matrix representing \hat{B} ?

The mirrors change the direction of the photon, or swap the eigenstates, so they are represented by the operator \hat{M}

$$\hat{M}|\phi_x\rangle = |\phi_y\rangle \quad (7)$$

$$\hat{M}|\phi_y\rangle = |\phi_x\rangle. \quad (8)$$

Question 2 In lab: Construct the matrix representing \hat{M} .

Because the arms of the interferometer have in general different lengths, we will create an operator for the propagation of the light through each arm of the interferometer. The “arm-propagating” operator will be:

$$\hat{A} = \begin{pmatrix} e^{i2\pi\ell_1/\lambda} & 0 \\ 0 & e^{i2\pi\ell_2/\lambda} \end{pmatrix}, \quad (9)$$

where ℓ_i is the length of distance traveled in arm i . The arm operator is unitary, so the state vectors are eigenstates of the arm-propagating operator.

The State of the light after the interferometer is

$$|\phi'_0\rangle = \hat{Z}|\phi_0\rangle \quad (10)$$

where

$$\hat{Z} = \hat{B}\hat{A}\hat{M}\hat{B} \quad (11)$$

Question 3 Take home: Find an expression for \hat{Z} (Mathematica is ideal for this).

Question 4 Take home: What is the wave-function of the photon past the second beam splitter? (Use matrix notation.)

What does this mean?

We start with the photon in a state. The first beam splitter puts the photon in a superposition of states. The second beam splitter puts the state of the light also into a superposition of output states. When we do a measurement the probability amplitude is not a constant number: the total probability amplitude for going to B is the superposition of the probability amplitudes for going through both arms. The probability of going to B shows *interference*. The interesting part of this is that in quantum mechanics superposition means that the path that the photon takes is undefined. If the photon is one whole thing how could that be? Note that this is not particular to photons. One could do superposition of electrons, atoms, etc. Scientists in Vienna have done interference experiments with buckyballs.

Question 5 Take home:

Find simplified expressions for b and c (see Eq. 1.)

When we put a detector in the direction x after the interferometer we are effectively applying the projector operator associated to path x :

$$\hat{P}_X = |\phi_x\rangle\langle\phi_x|. \quad (12)$$

Question 6 Take home: Find the matrix for \hat{P}_X .

Question 7 Take home: Show that the probability that the photon be at B is

$$P_B = b^*b = \frac{1}{2}(1 + \cos \delta), \quad (13)$$

where $\delta = 2\pi(\ell_1 - \ell_2)/\lambda$.

Question 8 In lab: What is the probability of the photon being in C after going through the interferometer? Consider the conservation of probability.

In summary, a consequence of superposition is *quantum interference*. By offering alternative paths, the interferometer naturally produces superposition. There is something different about the two paths: they could have different length. Thus if we vary the difference in lengths, we vary δ , and therefore the probability is going to oscillate between 0 and 1. In the lab we will change δ by changing ℓ_2 . We do this by pushing one of the mirrors very slightly (i.e., a few optical periods in very small increments). We accomplish this by means of a piezo-electric ceramic, which expands when a voltage is applied to it. The computer sends the voltage to the piezo in a stepwise manner so that we can scan δ .

2 Apparatus

The apparatus that we will use is shown in Fig. 2.

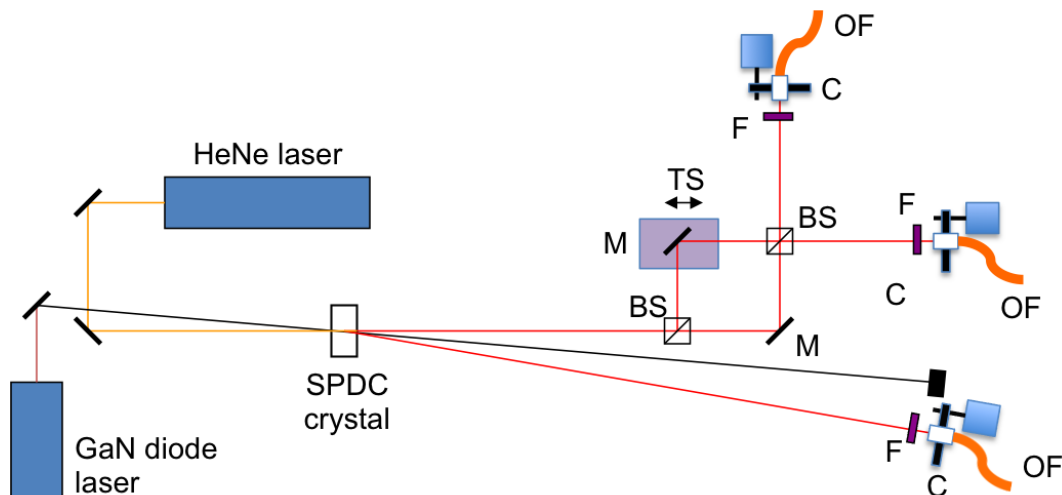


Figure 2: Apparatus to be used in the experiments. Optical elements of the interferometer are: beam splitters (BS), mirrors (M), with one of them mounted on a translation stage (TS). The photon-collection system has a 40-nm band-pass filter (F), a lens-collimator (C), and an optical fiber (OF). Photons are sent throughout the fiber to the detectors.

3 Procedure

The apparatus should be aligned from the previous lab. Once we are ready to detect single photons we turn the lights off and then we plug the detectors. Your tasks are the following:

1. Use the program “Continuouscounting” to verify that you are getting interference. Block one of the arms and maximize the counts by tilting the collimator for B.
2. Switch the electrical cables for B and C and do the same for the collimator C.
3. Flip the cables back and use the program “Quantum eraser” to take a scan of the piezo-electric that changes the phase of the interference program. Use the default values. The instructor will help you improve the visibility of the interference.
4. Now use the program “ContinuousCountingArduino.” In this program you can step the voltage of the piezo by hand on the program’s graphical interface. Change it so that you are at a minimum in the interference pattern. Block one of the arms with a card. What do you observe? Does it make sense? Can you explain it?

- Using the program “TripleContinuousCountingArduino” take a fast scan. It will show the counts at detectors A B and C, and the coincidences AB, AC and ABC. Once you are satisfied that it is all working, take a long scan of, say 3 s or more per point.

Question 9 Compare the graphs of AB, AC and reflect on the physics of the phenomenon you are observing. For the report show the two in a single graph. Remember to include error bars.

Question 10 What information do you get from the graph of ABC?

- Devise a new experiment: a measurement (scan or not) that illustrates the predictions of quantum mechanics or quantum physics in general.