Lab 4a The Quantum Eraser - Week 9

Phys434L Quantum Mechanics Lab
2018

March 26, 2018

1 Procedure for 3/26

1.1 Experiment: Aligning the components of the interferometer.

1. Using the HeNe laser place and align the waveplates into the arms of the interferometer.
2. Place the polarizer after the interferometer and put guides so that it can be put in and out without realignment.
3. Align the interferometer so that it shows no horizontal or vertical fringes (where do they arise?), but has one big fringe that appears or disappears with the changing phase.
4. With the help of the instructor align the interferometer so that it exhibits the least number of "white-light fringes." This step will involve a slight readjustment of the micrometer of the translation stage.

2 Larger Hilbert Spaces

2.1 Theory

A thorough description of the experiment is done using a formalism that combines the space of spatial directions, with the eigenstates $|\phi_x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\phi_y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and polarization, with the eigenstates $|H\rangle$ and $|V\rangle$, as shown earlier. The space of direction of propagation is two dimensional, a qubit; and the space of states of polarization is also two dimensional, another qubit. Thus we have 2 qubits. Quantum mechanics has a method to generate vectors and operator matrices of combined Hilbert spaces: it involves the tensor product, which is denoted by the symbol $\otimes$. In the tensor product operation, we multiply each element of one space (propagation direction) to each element of the other space (polarization). The ordering of spaces in the tensor product is important. In our case, we will order direction of propagation first, and polarization second.

For example, if we have a vector $|A\rangle$ in the space of propagation directions, and a vector $|B\rangle$ in the space of polarization, the tensor product of two vectors is:

$$|AB\rangle = |A\rangle \otimes |B\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \\ a_2 \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix}.$$

The eigenstates of our experiment are then:

$$|\phi_x, H\rangle = |\phi_x\rangle \otimes |H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$|\phi_x, V\rangle = |\phi_x\rangle \otimes |V\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

Question 1 In-lab: Find the vectors for $|\phi_y, H\rangle$ and $|\phi_y, V\rangle$.

The matrices for the operators in the larger space are the tensor product of the matrices of the operators that act on each space. For example, an operator in the direction of propagation space, $\hat{A}$, combines with an operator in the polarization space, $\hat{B}$, the following way:

$$\hat{A} \otimes \hat{B} = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \otimes \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} =$$

$$\begin{pmatrix} a_1 \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} & a_2 \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \\ a_3 \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} & a_4 \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_2 b_1 & a_2 b_2 \\ a_1 b_3 & a_1 b_4 & a_2 b_3 & a_2 b_4 \\ a_3 b_1 & a_3 b_2 & a_4 b_1 & a_4 b_2 \\ a_3 b_3 & a_3 b_4 & a_4 b_3 & a_4 b_4 \end{pmatrix}$$

Notice that the ordering procedure for the elements of the matrix is the same as for elements of the vectors.

Using the tensor product we can also construct the matrices for the interferometer. The beam splitter acts on one space and not the other, so it will be the tensor product of the beam-splitter matrix (first) with the identity (second). We put identity for the polarization part because the beam splitter does not alter the polarization. The matrix for the beam splitter in the larger space will be:

$$\hat{B}_2 = \hat{B}_1 \otimes \hat{1} = \begin{pmatrix} t & r \\ r & t \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} t & 0 & r & 0 \\ 0 & t & 0 & r \\ r & 0 & t & 0 \\ 0 & r & 0 & t \end{pmatrix},$$

where $t = 1/\sqrt{2}$ and $r = i/\sqrt{2}$.

Question 2 Take-home: Verify that the beam-splitter operator $\hat{B}_2$ acting $|\phi_x, V\rangle$ does not alter the polarization of the state.

Question 3 In-lab: Find the matrix $\hat{M}_2$ for the mirrors of the interferometer: $\hat{M}_1 \otimes \hat{1}$.

Question 4 Find the matrix for the interferometer phase: $\hat{A}_2$, where δ is the phase difference between the two arms, and:

$$\hat{A}_1 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}$$

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The quantum eraser has two wave plates in the arms of the interferometer. The matrix representing the half wave plate in the upper arm with angle θ and a half-wave plate with $\theta = 0$ in the lower arm is:

$$\hat{W}_2(\theta, 0) = \begin{pmatrix} \cos 2\theta & \sin 2\theta & 0 & 0 \\ \sin 2\theta & -\cos 2\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{1}$$

Question 5 Take-home: Find an expression for the full interferometer matrix (laborious): $\hat{Z} = \hat{B}_2 \hat{W}_2(\theta, 0) \hat{A}_2 \hat{M}_2 \hat{B}_2$. (You could use Matlab or Mathematica.)

Question 6 Take-home: Verify that when $\theta = \pi/4$

$$\hat{Z}_2 = \frac{1}{2} \begin{pmatrix} ie^{i\delta} & i & ie^{i\delta} & 1 \\ i & -ie^{i\delta} & 1 & e^{i\delta} \\ ie^{i\delta} & -1 & ie^{i\delta} & i \\ i & -e^{i\delta} & i & -ie^{i\delta} \end{pmatrix}, \tag{2}$$

Question 7 Take-home: Calculate the probability of photons in state $|\phi_x, V\rangle$, and entering the interferometer with the waveplate at $\theta = 0$, exit the interferometer in the same state.

Question 8 Take-home: Find the final state of the light when the wave plate is rotated an angle θ.

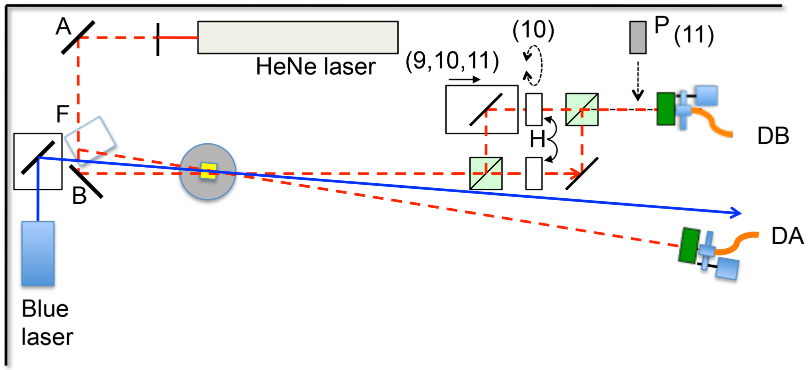

Figure 1: Schematic of the setup to recreate the quantum eraser. The numbers in parenthesis refer to the numbered steps in the procedure. Additional optical elements include half-wave plates (H) and a polarizer (P).

Question 9 Take-home: Find the probability of the photon leaving the interferometer in the x-direction and with polarization vertical, as a function of θ.

Question 10 In class: Find the probability that the photon leaving the interferometer in the x-direction and with polarization vertical, for $\theta = \pi/4$.

Question 11 In class: Find the probability that the photon leaving the interferometer in the x-direction and with polarization diagonal, for $\theta = \pi/4$.

1. Place half-wave plates in each arm of the interferometer, and set both to zero degrees. We use two half-wave plates because one alone would imbalance the optical path length of the two arms. One of them serves only to compensate for the added path length introduced by the other one. The full setup should look like the one in Fig. 1.
2. Do a piezo scan. It should show high-visibility fringes.
3. Rotate one of the waveplates by 45°. Redo the scan. It should show no fringes.
4. Place the polarizer (H) tilted 45° after the interferometer. Redo the scan. Fringes in DB should reappear at half the amplitude.