Lab 5 Delayed Choice - Week 11

Phys434L Quantum Mechanics Lab
2018

April 8, 2018

1 Theory

In 1983 John A. Wheeler, a famous theoretical physicist, proposed a thought experiment that introduced a striking aspect of quantum mechanics.[1] He called the concept *delayed choice*. In this lab we recreate a delayed-choice experiment with entangled photons. The experiment is not long, but has to be done very carefully.

1.1 The Set-up

In our previous experiments, one of the photons went through the interferometer, passed through a 40-nm filter, entered a 2-m optical fiber and reached a detector. The detector produced squared digital-electrical pulse per photon that was 3.3-V or 5-V in height and about 50-ns in duration. Each pulse traveled through a 2-m coaxial electrical cable arriving at a digital circuit that counted it and analyzed its coincident arrival with the pulse generated by the twin photon.

Because of the filter, the photons are in a superposition of energy states. Since we cannot know the exact energy of a photon going through the filter, the photons are in a superposition of energy states. They form a *wavepacket*. Earlier in the semester we discussed the concept of the coherence length, which we used to align the interferometers, and which we defined as the length of the photon wavepacket. If the bandwidth of the photon is $\Delta\lambda$, the coherence length is

$$\ell_c = \frac{\lambda^2}{\Delta\lambda}. \quad (1)$$

In our case, since $\lambda \simeq 810$ nm and $\Delta\lambda = 40$ nm, then $\ell_c = 16\ \mu$m.

[1] J.A. Wheeler, "Law Without Law," in *Quantum Theory and Measurement*, J.A. Wheeler and W.H. Zurek Eds. (Princeton University Press, 1983

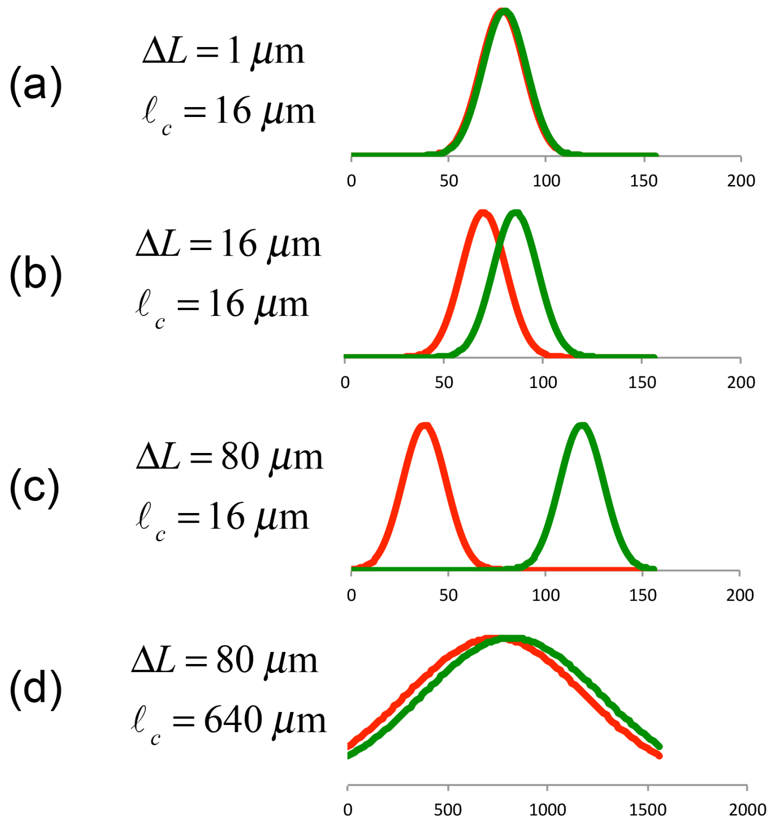

Figure 1: Comparison of Gaussian wavepackets of coherence length ℓ_c for the different path length difference ΔL in the interferometer.

1.2 The Plot

If the amplitude wavepackets for the light going through both paths overlap, then there is interference. The condition for this is

$$\Delta L < \ell_c. \tag{2}$$

If we see interference, we can say that we are observing the wave nature of light. The paths are indistinguishable and quantum mechanics argue that there is a superposition of amplitudes for the light going through the two possible paths. Figure 1(a) depicts the wavepackets with Gaussians.

Suppose now that we increase ΔL to be comparable or slightly larger than ℓ_c. In that case we will have partial knowledge of the path information. The wavepackets will not overlap exactly, so the interference will not be perfect. The wave functions for the two arms cannot not interfere completely. Thus, fringes will not show complete destructive or constructive interference. We quantify this via the visibility of the fringes, defined as

$$V = \frac{N_{\max} - N_{\min}}{N_{\max} + N_{\min}}, \tag{3}$$

where $N_{\max}$ and $N_{\min}$ are the maximum or minimum counts. When there is complete interference, $N_{\min} = 0$, and so $V = 1$. When there is no interference $N_{\max} = N_{\min}$, and so $V = 0$. Anything in between characterizes the partial knowledge of the path information. This situation is depicted in Fig. 1(b). As we increase the ΔL further, we see a decrease in the visibility of the fringes.

When $\Delta L \sim 80\ \mu\text{m} \gg \ell_c$ there is no temporal overlap between the wavepackets coming from the two arms of the interferometer (Fig. 1(c)). Thus, there is no interference, and $V \to 0$. We can think of this in an alternative way: the paths are distinguishable because we can do a timing measurement and find that traveling the shorter path results in an electronic detection at a distinguishably earlier time than when traveling the long path. This can be interpreted as the particle view: the knowledge of the path identifies the photons behaving as particles taking an individual path.

1.3 The Twist

The length of the wavepacket is determined by the bandwidth of the filter. What if we change the bandwidth of the filter to 1 nm? The length of the wavepacket will increase 40 times to 640 μm, so $\Delta L < \ell_c$. Notice the overlap of the wavepackets shown in Fig 1(d). Will there be interference? Yes.

The photons going through the interferometer are correlated in energy with their twin photons (remember that their energies have to add to the energy of the pump photon). The partner also has a filter. If we change the bandwidth of one filter, effectively means applying it to both, since we record the coincidences, or when both photons go through their respective filters. What then if instead of changing the filter on the photon that goes through the interferometer, we change the filter of the photon that does *not* go through the interferometer? It should not matter!

Moreover, there is no stipulation on when the photon should pass through the filter. Quantum mechanics does not care, which was the point of John Wheeler. So in this lab we will put this to the test. We will have the photon not going through the interferometer go through the 1-nm filter *well after* the other photon-the one going though the interferometer-has been detected. We will do this by sending the photon that does not go through the interferometer into a 20-m optical fiber. Because we detect the electronic signals of the two in coincidence we will add a corresponding length of electrical cable on the signal of the photon that goes through the interferometer. The schematic of the apparatus is shown in Fig. 2.

Should we see interference? The answer is a resounding yes! It seems like an illogical back action into the past: specifying the coherence length of the photon going through the interferometer after it is dead. Are we sending the terminator to kill Sarah Connor and make John Connor disappear?

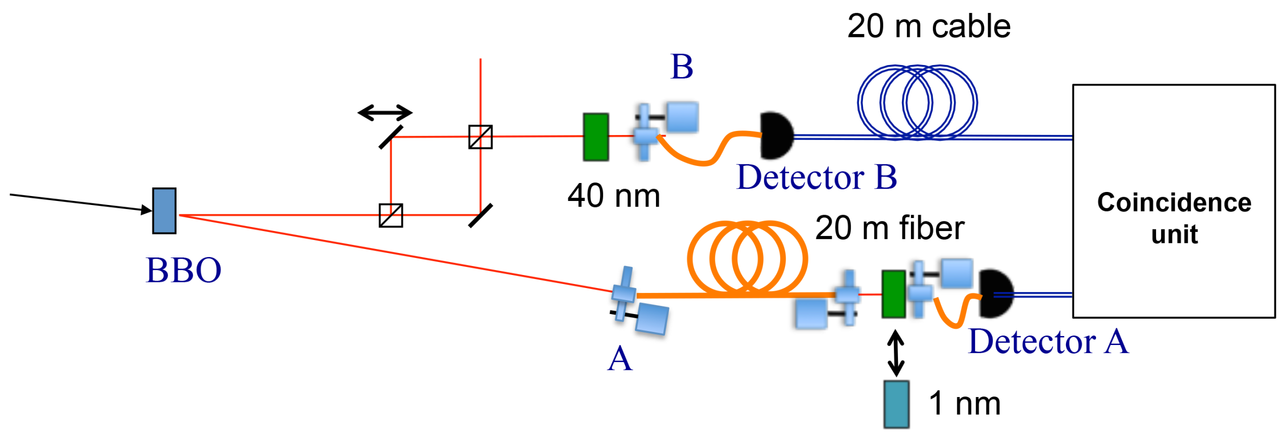

Figure 2: Apparatus for the Delayed-Choice Experiment

1.4 The Resolution

John Wheeler also said "a measured phenomenon is not a phenomenon until it is measured." For quantum mechanics there is no back action into the past. There is no past. The above analysis had a flaw. It did not consider the two photons for what they are: entangled in energy. Their wavefunction is a *two-photon* wavefunction:

$$|\psi\rangle = \int A(E)|E\rangle_1|E_0 - E\rangle_2 dE, \tag{4}$$

where $A(E)$ is the amplitude of producing a given pair of energies $(E, E_0 - E)$, and where E_0 is the energy of the pump beam. In the following, we show how quantum mechanics explains the experiment. The interferometer can be represented by the operator $\hat{U}$ acting on photon 1. If the coefficients of reflection and transmission of the beam splitters are r and t, and if the path-length difference of the arms of the interferometer is x, then the after the interferometer the state of the photons is

$$|\psi'\rangle = \hat{U}|\psi\rangle = \int A(E)rt(1 + e^{i2\pi Ex/hc})|E\rangle_1|E_0 - E\rangle_2 dE \tag{5}$$

We then pass the photons through two filters with transmission functions $a_1(E)$ and $a_2(E)$. Detectors following the filters complete an energy-projective measurement into states

$$|\psi'\rangle_i = \int a_i(E')|E'\rangle_i dE' \tag{6}$$

$i = 1, 2$. The detection probability is then given by

$$P = |\langle\psi'|_1\langle\psi''|_2|\psi\rangle|^2 . \tag{7}$$

Note that there is no time dependence. The final probability is independent of when the measurements are performed. The integral over the individual energy summations are reduced due to the orthogonality of energy eigenstates

$$\langle E'|E\rangle = \delta(E' - E), \tag{8}$$

giving rise to the following probability:

$$P = \left| \int a(E) rt (1 + e^{\mathrm{i}2\pi Ex/(hc)}) dE \right|^2 , \tag{9}$$

where $a(E) = A(E)a_1(E)a_2(E_0 - E)$. If we make, for simplicity that $a(E) = 1/(E_2 - E_1)$ when $E_1 \leq E \leq E_2$, and 0 otherwise then the integral is straightforward to solve giving

$$P = \frac{1}{(\Delta E)^2} \left| rt\Delta E + \mathrm{i}rt\frac{hc}{\pi x} e^{2\pi \overline{E} x/(hc)} \sin\left(\frac{\pi \Delta E x}{hc}\right) \right|^2 \tag{10}$$

where $\Delta E = E_2 - E_1$ and $\overline{E} = (E_1 + E_2)/2$. The above equation simplifies to

$$P = \frac{1}{4} \left[1 + \left(\frac{\sin\alpha}{\alpha} \right)^2 + 2\frac{\sin\alpha}{\alpha} \sin\left(\frac{2\pi \overline{E} x}{hc} \right) \right], \tag{11}$$

where $\alpha = \pi\Delta E x/(hc)$ and $rr^*tt^* = 1/4$. Note also that

$$\alpha = \frac{\pi x}{\ell_c} \tag{12}$$

Thus, when $x \ll \ell_c$ ($\Delta L \ll \ell_c$) we have $\sin\alpha/\alpha \to 1$, so the probability is

$$P = \frac{1}{2} \left[1 + \sin\left(\frac{2\pi x}{\overline{\lambda}} \right) \right], \tag{13}$$

with $\overline{\lambda} = hc/\overline{E}$, which corresponds to interference that varies with the path length difference x. As x increases to the limit when $x \gg \ell_c$ ($\Delta L \gg \ell_c$) , or $\alpha \to \infty$, then $\sin\alpha/\alpha \to 0$, the interference disappears and the probability becomes $P = 1/2$.

You can also skip the math. The bottom line is given by Eq. 7. Each detector collapses the state via a measurement. There is no restriction in either the time when the two projections are made nor their order. This is indeed not only "spooky action at a distance" but "spooky action over time." Discussions on this topic continue to this day. We ask: How can we change the pattern that we see after it has been generated? The wave or particle aspect is determined after the deed is done. The best answer that quantum mechanics tells us is that when the first detection is made, the data contains both wave an particle information. This experiment is a second recreation of the quantum eraser, but in this case we use time-energy projections to make the paths distinguishable or not. When we perform the second detection we select which subset of the data we choose to observe: wave or particle. In the words of Bohr: "If you are not disturbed by quantum mechanics, you do not understand it."

2 Parts

Qty	Part	Description/Comments
1	20-m fiber	Multimode.
1	20-m of coaxial cable	
1	Collimator mount	Mounted on a mirror mount
1	Collimator mount	From detector C in previous experiments
1	40-nm filter	Used for detector C
1	1-nm filter	To replace the 40-nm filter in C

3 Procedure

The instructor has verified that the apparatus is aligned from the previous lab. It is *very important* that the alignment of the interferometer is preserved.

1. With 40-nm filters for both photons take a scan that shows interference fringes. Find V for the data.

2. Nudge the micrometer of the interferometer and take a scan. Measure ΔL with the white-light spectrometer. Find V for the data.

3. Repeat the procedure above (nudge, scan, measure ΔL) until $\Delta L > \ell_c$ and you see no fringes.

4. Replace the 40-nm filter in A for the 1-nm filter. Retake the scan and measure V.

5. As a form of report, plot the data sets and one final plot of V vs. ΔL.

6. Explain the experiment and your findings.

7. Make concluding remarks about the significance of the experiment.