Lab 6a Polarization Entanglement - Week 12

Phys434L Quantum Mechanics Lab
2018

April 8, 2018

1 Theory

In this lab we are going to study the ultimate quantum mechanical object: two-particle entanglement. We have already worked with entangled states and manipulated them in the delayed-choice experiment. Here we will bring the topic of entanglement in a more direct way. It is ironic that Einstein in trying to prove quantum mechanics wrong thought out this marvelous situation, which Schrodinger later called entanglement. Two particles are entangled when their wavefunctions cannot be factorized. However, when we make a measurement on one particle, it instantaneously defines the state of the other particle. At least, that is quantum mechanics says...

1.1 State Production

In this lab we will create photon pairs entangled in polarization. If we denote the polarization states of a photon, horizontal and vertical, as $|H\rangle$ and $|V\rangle$, respectively, then the photon pairs that we created in previous labs would be in the state

$$|\psi_i\rangle = |V\rangle_1 |V\rangle_2. \tag{1}$$

This state is called a "product state" because the wavefunction of the pair is the product of the wavefunctions of the two particles.

The state of the two photons can be put in matrix form as

$$|\psi_i\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{2}$$

As we have seen before, a polarizer projects the state of the light into the direction of the transmission axis of the polarizer. If the polarizer is rotated an angle θ relative to the horizontal, then the polarizer eigenstates of transmission are

$$|\theta\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{3}$$

Thus, if we decide to measure the two photons with polarizers set to angles θ_1 and θ_2, as shown in Fig 1, then effectively the photons are projected into the state

$$|\psi_p\rangle = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \otimes \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_1\cos\theta_2 \\ \cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 \\ \sin\theta_1\sin\theta_2 \end{pmatrix}. \tag{4}$$

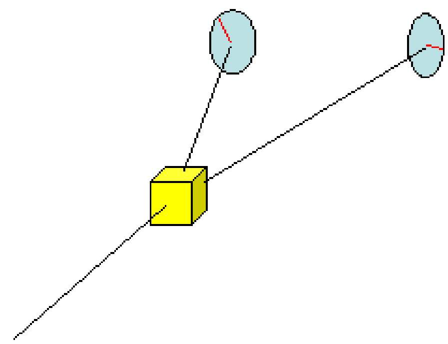

Figure 1: Down-conversion production of photons pairs and their detection with polarizers.

The probability of joint detection (i.e., of detecting both photons past the polarizers) is the square of the probability amplitude of being in state $|\phi_p\rangle$

$$P_p = |\langle\psi_p|\psi_i\rangle|^2. \tag{5}$$

Question 1 Find an expression for P_p

The polarization state of the down-converted light that we created in previous labs was due to a pump beam that was horizontally polarized incident on an appropriately oriented crystal. By adjusting the crystal we can have down-converted pairs at 804 nm come out forming an angle with the incident direction, as shown in Fig 1. If we change the polarization of the pump to vertical we would not get down-conversion. However, if we rotate the crystal by 90° we get horizontally polarized pairs.

Question 2 Find the joint detection probability P_p past the polarizers when the initial state is $|\psi_i\rangle = |H\rangle_1|H\rangle_2$.

A few years ago Paul Kwiat (U. Illinois) came up with a clever trick: to put two thin down-conversion crystals back to back but rotated by 90° with respect to each other. He then sent a pump beam polarized at 45° to the pair of crystals, as shown in Fig. 2. This way the horizontal component of the pump polarization produces vertically polarized pairs with one crystal and the vertical component produces horizontally polarized pairs with the other

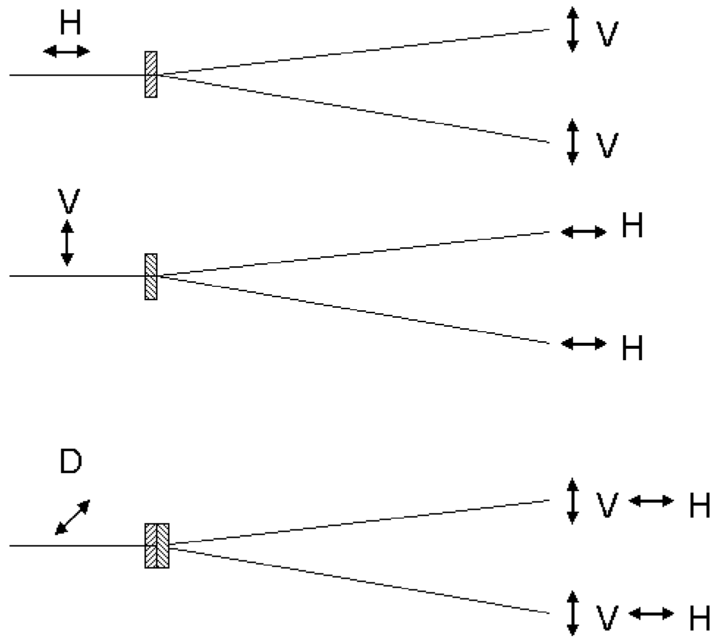

Figure 2: Method to produce polarization-entangled states: the bottom setup is a superposition of the two cases above.

crystal. If crystal thickness is smaller than the beam width, then there is no way to tell in which crystal the photon pairs were created. Thus when the paths are indistinguishable the photon pairs get created into a state that is a superposition of the two possibilities:

$$|\Phi_{ent}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2 e^{i\delta}\right), \tag{6}$$

where δ is a phase between the two possibilities. For simplicity, and without much loss of generality let us assume that $\delta = 0$. The state is then

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \tag{7}$$

The skeptical physicist may say: "What about a mixed state?" A mixed state would be the situation where the light is not in a superposition of both horizontal and both vertical. Rather, half the time the pairs come horizontal and half the time they come vertical. How can we distinguish the two? The answer to this questions leads us straight to Bell.

The Dirac notation cannot describe a mixed state so we need to resort to another artifact of quantum theory: the density matrix. For a pure state $|\psi\rangle$ the density matrix is defined as

$$\hat{\rho}_\psi = |\psi\rangle\langle\psi|. \tag{8}$$

That is, it is the outer product of the state vector with itself. As an example consider the state $|\Phi^+\rangle$. It is given in matrix form by

$$\hat{\rho}_{\Phi^+} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

Before we discuss Bell let us study in more detail the entangled state given by Eq. 7. What would the form of the state be in the diagonal-antidiagonal basis? If you recall, the diagonal basis states are related to the horizontal-vertical states by:

$$\begin{aligned} |D\rangle &= \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right) &(10)\\ |A\rangle &= \frac{1}{\sqrt{2}}\left(-|H\rangle + |V\rangle\right). &(11)\end{aligned}$$

If we put $|H\rangle$ and $|V\rangle$ in terms of $|D\rangle$ and $|A\rangle$ for each particle in Eq. 7, we can show that

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle_1|D\rangle_2 + |A\rangle_1|A\rangle_2\right). \tag{12}$$

In the H-V basis the photons are in a superposition of being parallel to each other in two different ways. In the rotated basis they are also in an entangled state that is a superposition of the two possibilities in which they are parallel! This is an interesting but unique aspect of state $|\Phi^+\rangle$. Let us verify this the following way

$$\begin{aligned} |\Phi^+\rangle &= \frac{1}{\sqrt{2}}\left(|D\rangle_1|D\rangle_2 + |A\rangle_1|A\rangle_2\right) &(13)\\ &= \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}. &(14)\end{aligned}$$

State $|\Phi^-\rangle$ corresponds to state $|\Phi_{ent}\rangle$ of Eq. 6 with $\delta = \pi$. In the diagonal basis $|\Phi^-\rangle$ becomes

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle_1|A\rangle_2 + |A\rangle_1|D\rangle_2\right). \tag{15}$$

That is, in state $|\Phi^-\rangle$ the light switches from being parallel in the H-V basis to being orthogonal in the D-A basis.

Let us go back to $|\Phi^+\rangle$. Suppose that we send both photons through two polarizers. The density matrix for the state is a bit ominous if you apply Eq. 8 to the state of Eq. 4. The probability that a photon in a state ϕ is measured to be in state ψ (the absolute-value squared of the inner product) is given by the trace of the product of the two density matrices. The trace is the sum of the diagonal elements of the product matrix:

$$P_{\phi,\psi} = |\langle\psi|\phi\rangle|^2 = \mathrm{Tr}(\hat{\rho}_\psi\hat{\rho}_\phi) \tag{16}$$

Question 3 Find the density matrices for the state ψ_p when a) $\theta_1 = \pi/4$ and $\theta_2 = \pi/4$, and b) $\theta_1 = \pi/4$ and $\theta_2 = -\pi/4$.

Question 4 Calculate the probability of detecting entangled state with polarizers aligned to angles a) $\theta_1 = \pi/4$ and $\theta_2 = \pi/4$, and b) $\theta_1 = \pi/4$ and $\theta_2 = -\pi/4$.

The above probabilities are the ones predicted by entanglement. You can see that the detection of one is correlated with the result of the detection of the other one. It is laborious, but can be shown that

$$P_{\psi_p} = \frac{1}{2}\cos^2(\theta_1 - \theta_2) \tag{17}$$

In inspecting Eq. 17 you can see that we get maximum probability when the two angles are the same. That is, the photon pairs in state $|\Phi^+\rangle$ are parallel in *any* basis. As soon as we measure one of the photons to be polarized along one direction (with probability 1/2) we get that the other one is polarized in the same direction with unity probability. Thus, one can think of the "1/2" in Eq. 17 as the probability of detecting the first photon, and the cosine term as the conditional probability of detecting the other given that the first one was detected at the angle θ_1. This correlation is the basis for *nonlocality*; that the detection of one photon immediately "collapses" the wavefunction of the two, instantaneously at faster than the speed of light. This is the view advocated by Bohr in the so called "Copenhagen interpretation" of quantum mechanics, which Einstein criticized and derided as "spooky action at a distance."

One last point. The "1/2" term stems from the randomness of quantum mechanics. When we detect the polarization of the first photon anything can happen. It may be transmitted or not. It is the detection of the second one that is conditional to the *result* of the first detection. Thus, this is not faster-than-light communication of *information*, because we do not control the outcome of the first measurement.

How do we know that we are not in a mixed state? This is the "realistic view." In this view the photons had their state of polarization defined before the measurement was done. The mixed state is the probabilistic combination of two or more pure states. For the case of the mixture of state $|\psi\rangle$ with probability $\mathcal{P}_\psi$ and state $|\phi\rangle$ with probability $\mathcal{P}_\phi$, with $\mathcal{P}_\psi + \mathcal{P}_\phi = 1$,

$$\hat{\rho}_m = \mathcal{P}_\psi|\psi\rangle\langle\psi| + \mathcal{P}_\phi|\phi\rangle\langle\phi| \tag{18}$$

As an example, let us find the mixed state mentioned earlier: the realistic alternative to the $|\Phi^+\rangle$. It entails the product states each with a probability of 1/2

$$\hat{\rho}_{HH,VV} = \frac{1}{2}|H\rangle_1|H\rangle_2\langle H|_1\langle H|_2 + \frac{1}{2}|V\rangle_1|V\rangle_2\langle V|_1\langle V|_2 \tag{19}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{20}$$

Note that this matrix is different than the one of Eq. 9, which means that we can use it to tell the difference between the two situations.

Question 5 Calculate the probability of detecting photons in a mixed state with polarizers aligned to angles a) $\theta_1 = \pi/4$ and $\theta_2 = \pi/4$, and b) $\theta_1 = \pi/4$ and $\theta_2 = -\pi/4$.

The probability of detection predicted for the mixed state is given by

$$P_{\text{mix}} = \frac{1}{2}\cos^2\theta_1\cos^2\theta_2 + \frac{1}{2}\sin^2\theta_1\sin^2\theta_2. \tag{21}$$

The one half represents the situation that the light is in either state, $|H\rangle_1|H\rangle_2$ or $|V\rangle_1|V\rangle_2$ half the time. Notice that Eqs. 17 and 21 have a different functional form. Thus, we have a chance to find out which one is correct. When $\theta_2 = 0$ both give the same answer:

$$P_{\text{ent}} = P_{\text{mix}} = (1/2)\cos^2\theta_1.$$

However, if $\theta_2 = \pi/4$ they give a different answer:

$$P_{\text{ent}} = (1/2)\cos^2(\theta_1 - \pi/4),$$

while

$$P_{\text{mix}} = 1/4.$$

In this lab we will do this measurement.

One final comment. There are two other states that also follow similar results. These are described by the state

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2\right). \tag{22}$$

The four set of states $|\Phi^{\pm}\rangle$ and $|\Psi^{\pm}\rangle$ are also known as the "Bell" states. They have interesting properties too: photons in state $|\Psi^{-}\rangle$ are in a superposition of possible ways of being orthogonal in *any* basis, while photons in state $|\Psi^{+}\rangle$ are orthogonal in the H-V basis and parallel in the D-A basis.

1.2 Parts

This experiment entails 2 important modifications of the previous setup: changing the down-conversion crystal and removing the interferometer. The latter entails realigning the collimator for detector B. A few optical components will also need to be added. The parts that are needed are listed in the table below and refer to Fig. 3

Qty	Part	Description/Comments
1	BBO crystal pair	(X) Two type-I BBO crystals 0.5 m thick, with 29° phase matching rotated 90° relative to each other and mounted on a rotation plus tilting (mirror-like) mount. Axes of the crystals must be in horizontal/vertical planes.
1	Half-wave plate	(V) 405 nm, zero order.
1	Compensating crystal	(U) Quartz, A-cut, 6-8 mm thick on a rotation mount.
2	Half-wave plate	(H) 810 nm, zero order.
2	Prism polarizer	(P) Glan-Thompson.

1.3 Procedure

The procedure involves the following steps:

1. Set up the apparatus shown in Fig. 3. Replace the crystal for the new one exactly in the same place, and aligned as perpendicular to the pump beam as possible.

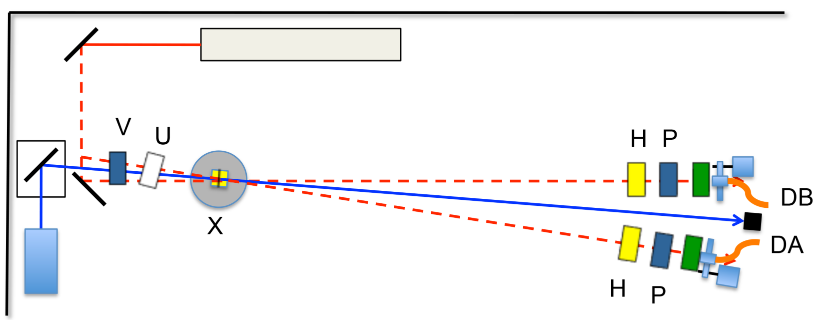

Figure 3: Apparatus for measuring entanglement and Bell Inequalities. It includes a half-wave plate and quartz crystal (U) for the pump beam (V), and half-wave plates (H) and polarizers (P) for the down-converted photons. Down-conversion crystals are (X).

2. Polarizers (P) must be set to horizontal transmission. Half-wave plates (H) must be set to 45°, and half-wave plate (V) must be set to 0. This is set up to produce and detect down-conversion photons that are vertically polarized.

3. Start the apparatus and tilt the crystal (X) about a vertical axis to maximize coincidences. A sign that pairs are being detected is that they have the same polarization. Thus if you set one of the half-wave plates (H) to detect horizontally polarized photons (What angle would that be?), you should not detect any coincidences.

4. Set the half-wave plate (V) so that the pump beam is vertically polarized. Set the half-wave plates (H) to make sure horizontally-polarized photons are produced.

5. Set the half wave plate V to about 22.5 degrees. By setting to measure both photons horizontally polarized and then both vertically polarized, adjust the half-wave plate so that the coincidences are the same for both cases.

6. Set to detect photon 1 in state D and photon 2 in state A. Adjust the tilt of the crystal U to minimize the coincidences.

   In a previous section we mentioned that when we produce the pairs, they are in an entangled state because we cannot distinguish from which crystal they were produced. However, the production in different crystals is not symmetric: in one case the pump beam creates pairs in the first crystal and the down converted photons travel with a speed c/n^*_{dc} through the other crystal (c is the speed of light and n^*_{dc} is group index of refraction); in the other case the pump photon travels through the first crystal at a speed c/n^*_p and the pairs are generated in the second crystal (n^*_p is group index of refraction). In the crystal where the pairs are generated, the speeds are the same, but in the crystal where they are not generated they travel at different speeds because $n^*_p > n^*_{dc}$. If we imagine the photons are wave-packets or pulses, the above difference amounts to a time delay, which introduces distinguishability and reduces the fidelity of the state. Moreover, this difference also translates into a phase shift. This leaves a phase shift δ in the state:

$$|\psi'\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2 e^{i\delta} \right) . \tag{23}$$

   We mentioned earlier the correlations that are expected of the state when $\delta = 0$: when the polarizers are set to detect one photon in state $|D\rangle$ and the other photon in state $|A\rangle$, we should get no coincidences. However, if $\delta \neq 0$ we can get coincidences. Worse yet, if $\delta = \pi$ we get that $P_{DA/\psi'} = 1/2$ (a maximum). Thus, we must adjust δ to be 0. The quartz crystal, oriented correctly compensates for the time delay by pre-delaying in time one polarization component of the pump beam relative to the other. It also introduces an additional phase shift. We can adjust δ via the phase shift introduced by the quartz crystal, by tilting of the quartz crystal (U). When this task is completed, we have the entangled state and are done for the day.