Lab 6b Bell Inequality - Week 13

Phys434L Quantum Mechanics Lab
2018

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1 Theory

This final experiment is a fundamental one in quantum physics. It is not your typical physics lab where you measure something new or confirm a physical theory. Surely it has both those aspects of laboratory experimentation, but this experiment also settles a philosophical debate. Is nature realistic? That is, do objects have an inherent reality? Is a frog a frog? You would answer: "Of course it is!" Quantum mechanics says that too, but only because the frog has billions of atoms interacting with their surroundings. Suppose we settle for something simpler, say a single photon, and launch it by turning-on a specially prepared source. The flying photon has definite properties: energy, momentum, polarization, right? Well, that might not be strictly true. Quantum mechanics allows for superposition. A photon can be in a superposition of having two (or more) energies, say red and blue, or polarizations, horizontal and vertical. It can do so as long as the two possibilities are indistinguishable. That is, that once the photon is launched, it is impossible to distinguish those two properties. However, when we measure the property by something that distinguishes them, red from blue with a filter, or horizontal from vertical with a polarizer, the state of the photon snaps, randomly, into one of the possibilities. We measure this by doing repeated measurements of identical photons launched at different times. This is in contrast to what a realistic view would say: when the photons are launched they come out in a statistical mixture of being red or blue, or horizontal or vertical, but all along, once launched, the properties of the photon are the same and definite. Quantum mechanics says something very different: the reality of the color or the polarization of the photon is undefined until it is measured.

Let us take our argument one step further. Let us consider two photons, which can be horizontal or vertical, and launch them in a state where of both are vertically polarized in a superposition of both being horizontally polarized. We can prepare them in two ways: both being vertical and both being horizontal. If we make them such that the two possibilities are indistinguishable, you are in such a superposition: two polarizations at the same time. The state is given by

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2 \right). \tag{1}$$

Then you send both photons in distinct directions. Quantum mechanics says that the measuring the polarization of one snaps the state of both into one of the possibilities, randomly. More strikingly, the polarization of the second photon, somewhere else far away, snaps into the same state, instantaneously! So which is it? Are the polarizations of the photons parallel but defined all along before the measurement, or undefined until the measurement is made?

This argument was of much debate since 1935, but considered only academic. That all changed in 1964 when John Bell devised a situation where these two distinct philosophical possibilities could be resolved by an experiment. Experimenters quickly set out to do the experiment. Today, after a long series of landmark experiments, the issue is all settled: quantum mechanics is correct. Nature is not realistic, and also "nonlocal" (the instantaneous snapping of the state of the other photon, or "spooky action at a distance"). However, it is stunning that one can do an experiment that tests this fundamental question. Our experiment here does precisely that: you will confirm that nature is not-realistic and nonlocal. We will do this with photons and their polarization.

One of the versions of Bell inequalities we present here goes by the name Clauser-Horne-Shimony-Holt (CHSH) inequality, after the names of the authors. We begin by defining a variable E that expresses the correlation between the polarizations of the two photons. If they are perfectly correlated, as is the case in state $|\psi\rangle$, then $E = 1$. If they are uncorrelated then $E = 0$. The expectation value of the correlation can be defined for arbitrary angles θ_1 and θ_2. At these angles E is given by

$$E(\theta_1, \theta_2) = P(\theta_1, \theta_2) + P(\theta_1 + \frac{\pi}{2}, \theta_2 + \frac{\pi}{2}) - P(\theta_1, \theta_2 + \frac{\pi}{2}) - P(\theta_1 + \frac{\pi}{2}, \theta_2), \tag{2}$$

For state $|\psi\rangle$ it can be shown that the expression for E reduces to

$$E_\psi(\theta_1, \theta_2) = \cos[2(\theta_1 - \theta_2)] \tag{3}$$

The perfect correlation of the state $|\psi\rangle$ is manifested in Eq. 3 because $E(\theta, \theta) = 1$ regardless of θ. In contrast, the mixed state gives a different correlation:

$$E_{\text{mix}} = \cos(2\theta_1)\cos(2\theta_2). \tag{4}$$

Therefore, one can find situations where the two expectation values give different results.

Question 1 Take-home: Calculate the correlation parameters $E(0, 0)$ and $E(\pi/4, \pi/4)$ for both the entangled state and the mixed state. State your conclusions about the results.

Note that to obtain E we need to do measurements pairwise at the combinations of the 4 angles: θ_1, θ_2, $\theta_1 + \pi/2$ and $\theta_2 + \pi/2$. Furthermore, CHSH defined a variable S that depends on 4 angles, two for each photon. It is given by

$$S = E(\theta_1, \theta_2) - E(\theta_1, \theta_2') + E(\theta_1', \theta_2) + E(\theta_1', \theta_2'), \tag{5}$$

where θ_1 and θ_1' are two polarizer angles for photon 1, and similarly, θ_2 and θ_2' are two angles for photon 2. Given that the measurements of E require the orthogonal angles too, then to get a value for S we need to make 16 pairwise measurements. A local realistic theory *satisfies*

$$|S| \leq 2. \tag{6}$$

Entangled states measured at certain angles will violate this inequality.

Question 2 Take-home: Show that the inequality of Eq. 6 is violated for state $|\psi\rangle$ when $\theta_1 = -45°$, $\theta_1' = 0$, $\theta_2 = -\pi/8$ and $\theta_2' = \pi/8$.

Question 3 Take-home: Show that the inequality of Eq. 6 with E_{mix} of Eq. 4 is satisfied when $\theta_1 = -45°$, $\theta_1' = 0$, $\theta_2 = -\pi/8$ and $\theta_2' = \pi/8$.

The measurement of the parameter S that provides the maximal violation of the Bell inequality of Eq. 6, can be done for state of Eq. 1 with the angles: $\theta_1 = 0°$, $\theta_1' = 45°$, $\theta_2 = 22.5°$, $\theta_2' = 67.5°$. Because of a lack of 100% detector efficiency the correlation parameters have to be normalized as follows:

$$E(\theta_1, \theta_2) = \frac{N(\theta_1, \theta_2) + N(\theta_1 + \frac{\pi}{2}, \theta_2 + \frac{\pi}{2}) - N(\theta_1, \theta_2 + \frac{\pi}{2}) - N(\theta_1 + \frac{\pi}{2}, \theta_2)}{N(\theta_1, \theta_2) + N(\theta_1 + \frac{\pi}{2}, \theta_2 + \frac{\pi}{2}) + N(\theta_1, \theta_2 + \frac{\pi}{2}) + N(\theta_1 + \frac{\pi}{2}, \theta_2)}, \tag{7}$$

where $N(\theta_1, \theta_2)$ is the number of coincidences in a set amount of time when the polarizer angles are θ_1 and θ_2.

An important quantity is the uncertainty in S, given by

$$\Delta S = \sqrt{\sum_{i=1}^{16} \left(\Delta N_i \frac{\partial S}{\partial N_i} \right)^2}, \tag{8}$$

where the uncertainties in the counts are due to statistical errors: $\Delta N_i = \sqrt{N_i}$.

1.1 Procedure

The procedure involves the following steps:

1. Last week you set up the apparatus shown in Fig. 1. Take preliminary measurements to verify that the setup is working properly per last week's alignments.

2. Take measurements for 5 s by appropriately adjusting HWP_1 and HWP_2 to fill-in the table below.

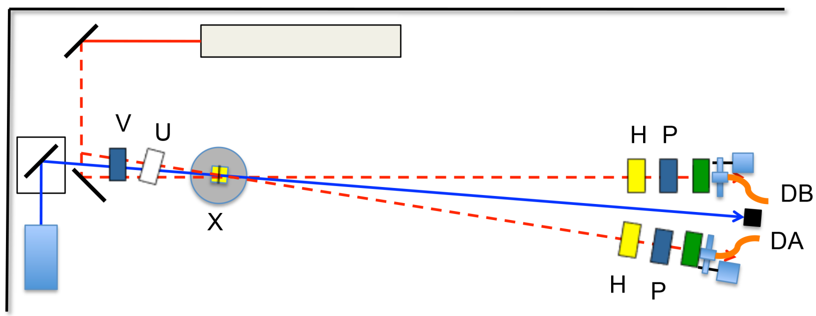

Figure 1: Apparatus for measuring entanglement and Bell Inequalities. It includes a half-wave plate and quartz crystal (U) for the pump beam (V), and half-wave plates (H) and polarizers (P) for the down-converted photons. Down-conversion crystals are (X).

θ_1	θ_2	HWP_1	HWP_2	$N(\theta_1, \theta_2)$
0	22.5			
45	22.5			
90	22.5			
135	22.5			
0	67.5			
45	67.5			
90	67.5			
135	67.5			
0	112.5			
45	112.5			
90	112.5			
135	112.5			
0	157.5			
45	157.5			
90	157.5			
135	157.5			

3. Calculate S by entering the values in the spreadsheet that is provided.
4. Calculate the error in your determination of S. State your conclusions.
5. Rotate the quartz plate (U) by 90 degrees. This should put the light into a mixed state. Retake the data and evaluate S for this situation.
6. Write a conclusion based on your results.