Chapter 19

Photons and Quantum Mechanics

19.1 Interference of quanta

You have seen that light sometimes behaves like a wave and sometimes like a particle. When laser light passes through slits it produces interference fringes. When the intensity is cut down significantly we start to reach the limit when light is made of small lumps, photons. Reducing the intensity further only reduces the frequency at which these come, but not the size of the lumps.

Exercise 1 A "bright" source of single photons sends about 10^6 photons per second. If these photons are evenly separated in a stream, what is the distance between photons? (Ans: 300 m)

A fundamental principle of quantum mechanics is superposition. If an event can occur in several alternate ways that are indistinguishable, then the probability amplitude for such an event is the superposition of the probability amplitudes for each way. Note however, that by this quantum mechanics means that the event occurs in *all ways simultaneously*.

Feynman summarized the quantum mechanics of alternate paths in three simple rules:[1]

1. The probability of an event in an ideal experiment is given by the square of the absolute value of a complex number ϕ which is called the probability amplitude:
$$P = |\phi|^2, \tag{19.1}$$

2. When an event can occur in several alternative ways, the probability amplitude for some event is the sum of the probability amplitudes for each way considered separately:
$$P = |\phi_1 + \phi_2|^2, \tag{19.2}$$

[1] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, 1965) V. 3, p. 1-1.

3. When an experiment is performed which is capable of determining whether one or another alternative path is actually taken, the probability of the event is the sum of the probabilities for each alternative:

$$P = P_1 + P_2 = |\phi_1|^2 + |\phi_2|^2. \tag{19.3}$$

In the case of a photon going through the Mach-Zehnder interferometer shown in Fig. 19.1, if the paths are indistinguishable the probability amplitude in going from A to B is the superposition of the probability amplitudes in going through each arm, ϕ_1 and ϕ_2.

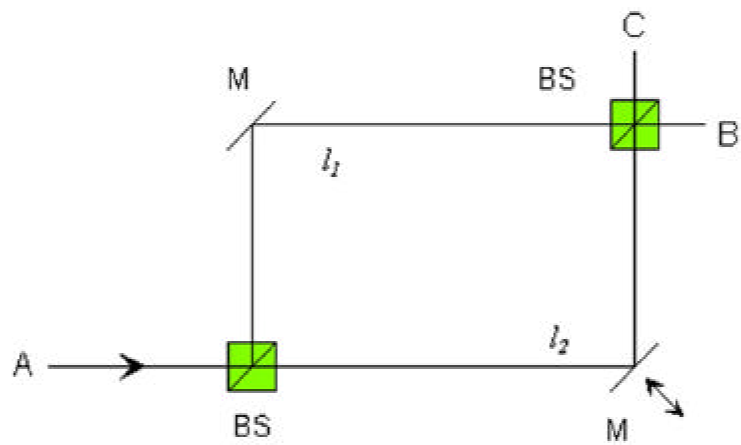

Figure 19.1: Diagram of a Mach-Zehnder interferometer.

The beam-splitters (BS) used in the interferometer reflect half of the intensity and transmit half of the intensity. Thus the probability of a photon being reflected or transmitted is 1/2. Since the probability is the square of the probability amplitude, then the probability amplitude of being reflected or transmitted is $1/\sqrt{2}$.

Suppose that we block arm 2. The probability amplitude of the photon being reflected at the first beam splitter is $1/\sqrt{2}$. The probability amplitude of transmitting at the second beam splitter is also $1/\sqrt{2}$. Thus the probability amplitude in going from A to B has an absolute value that is the product of the two probability amplitudes, or $|\phi_1| = 1/2$. The probability of going from A to B when arm 2 is blocked is then $P_1 = |\phi_1|^2 = 1/4$. The probability amplitude ϕ_1 also has a phase δ_1 due to path 1.

Exercise 2 What is the probability for a photon to go from A to C when arm 2 is blocked? (Ans: 1/4)

If both arms are unblocked then we have two probability amplitudes. If the paths are indistinguishable, then according to Feynman's second rule we represent the resulting superposition mathematically by summing the two amplitudes:

$$\phi = \phi_1 + \phi_2 \tag{19.4}$$

We note that this is not a simple sum because ϕ_1 and ϕ_2 are not simple numbers. The probability of the event (i.e., a photon going from A to B) is given by

$$P = |\phi|^2 = |\phi_1|^2 + |\phi_2|^2 + 2|\phi_1||\phi_2| \cos \delta \tag{19.5}$$

We can understand Eq. 19.5 by viewing the probability amplitudes as "clock vectors," where the length of the clock arm is the absolute value of the probability amplitude, $|\phi|$, and the orientation of the clock vector δ represents the phase. In the case of the two paths the amplitudes are $|\phi_1|$ and $|\phi_2|$, and the phases are $\delta_1 = 2\pi/\lambda$ and $\delta_2\delta_2 = 2\pi/\lambda$.

The sum of the two amplitudes ϕ is done mathematically as shown in Fig. 19.1. We form a triangle by connecting the two vectors consecutively one after the other. The final probability amplitude is obtained by joining the beginning and the end of the two vectors arranged consecutively, as shown in Fig. 19.1.

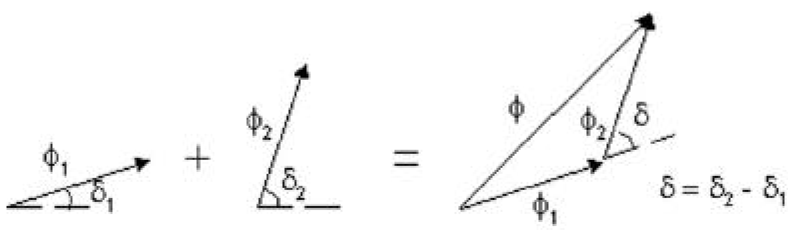

Figure 19.2: Method to add the probability amplitudes.

The absolute value of ϕ is obtained using the law of cosines, which if you look carefully, that is indeed what Eq. 19.5 is. In the case of our Mach-Zehnder interferometer we have that $|\phi_1| = |\phi_2| = 1/2$. Replacing this relation into Eq. 19.5 and simplifying, we get

$$P = \frac{1}{2}(1 + \cos \delta) \tag{19.6}$$

Notice that when δ is a multiple of 2π the probability of a photon going from A to B is 1. That means that *every* photon that enters the interferometer ends up in B. Conversely, if δ is an odd multiple of π then $P = 0$. No photon reaches B. Where do the photons go?

Exercise 3 If the probability of going from A to B is given by Eq. 19.6, what is the probability for a photon to go from A to C? Hint: conservation of energy. (Ans: $(1/2)(1 - \cos \delta)$)

We can do an experiment with photons by sending a very weak beam of light into the interferometer. At B we put a detector that detects one photon at a time. After it absorbs a photon the detector sends an electronic pulse to a counter. If we sent N photons in one second, then the detector will record NP photons in one second, where P is the probability given by Eq. 19.6. The Graph of Fig. 19.1 shows an experimental recording of the number of counts detected by a photon detector in 2 s as a function of a variable that is proportional to the phase δ.

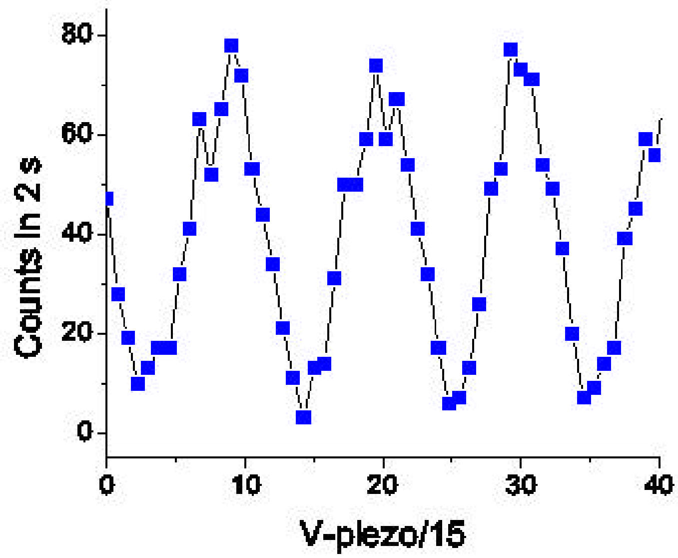

Figure 19.3: Photon counts recorded by a detector at B.

The variable is the voltage applied to a piezo-electric element that displaces one of the mirrors of the interferometer. This displacement in turn changes the length of one of the arms, and as a consequence, δ.

Exercise 4 Assuming that the detector records every photon incident on it (in practice this is not the case), what is the number of photons incident to the interferometer in 2 s? (Ans: 80)

Exercise 5 Following Fig. 19.1, what graph would you expect to get if the paths were distinguishable? (Ans: N 40)

In summary, when quantum mechanics states that the photon is in a superposition of taking the two paths, it is meant that the photon takes both paths. Another way to put it is that the photon interferes with itself. More strikingly, quantum mechanics says that when the intensity is increased so that many photons, billions of billions, pass through the interferometer at the same time, interference is produced by *each* photon interfering with itself!

If we decide to determine which path the photon takes by means of measurement, then the photon paths are not indistinguishable and the photon is no longer in a superposition of the two paths. When we do a measurement of the path we find that the photon takes either one path or the other. When the paths are distinguishable the interference disappears.

The most straight forward way to make the paths distinguishable is by putting a detector in one of the arms of the interferometer, say arm 2. When we do this then there is no interference because obviously the light is not going through arm 2. But there is an interesting experiment that we can perform that distinguishes the paths without blocking the light. In the next sections we will discuss a property of photons, polarization, that can be used to distinguish the paths.

19.1.1 Does the photon split?

It is possible to do an experiment that tests whether the photon splits when going through the two arms of the interferometer. [2] Consider the setup of Fig. 19.4. We have a source that emits pairs of photons simultaneously. The

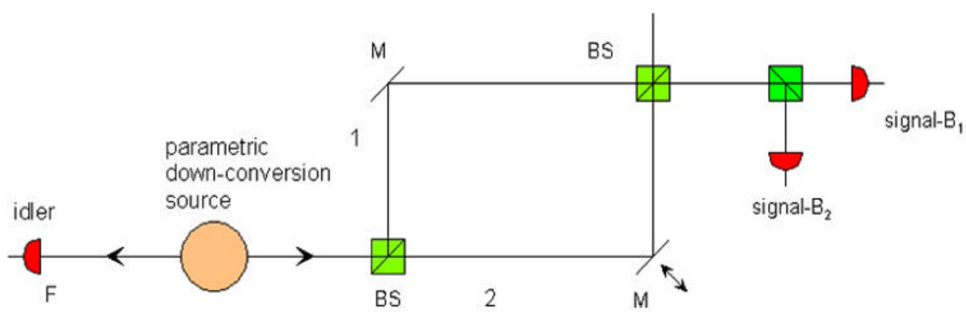

Figure 19.4: Diagram of a tagged photon experiment on interference with a Mach-Zehnder interferometer.

process is a well known optical effect called "parametric down-conversion." In this process one photon incident on a crystal is converted into two photons with energies that add up to the energy of the parent photon. The source is very weak (i.e., produces about 10^6 photons per second), but that is what we want, since we wish to detect the interference of one photon with itself. The data obtained in Fig. 19.1 was obtained this way.

In this method we send one photon, called the "idler," to a detector, and the other photon, called the "signal," to an interferometer and then to a detector. The interesting thing about this source is that by only detecting the electronic signals of photons that arrive simultaneously at the detectors, we discriminate against other photons that we do not know about. We only detect photons that have a partner. Thus, we "know" that we are detecting what happens to a lone signal photon going through the interferometer. It serves to underscore that the photon interferes with itself.

But the endeavors of the photon get even more mysterious. If we accept that the photon goes through "both arms" then that means that somehow it splits. Does it? Well, our setup in Fig. 19.4 was made with the following rationale: if the photons split at one beam splitter, then they split at other ones. Therefore, we put a beam splitter after the interferometer and put a detector after it. Under these circumstances now we have two detectors after the interferometer, which we call "signal-B_1" and "signal-B_2."

[2] E.J. Galvez, C.H. Holbrow, M.J. Pysher, J.W. Martin, N. Courtemanche, L. Heilig, and J. Spencer, "Interference with Correlated Photons: Five Quantum Mechanics Experiments for Undergraduates, American Journal of Physics (in press)

Now comes the photon-splitting test. We arrange our electronic signals so that we detect the double coincidences between the idler detector and signal-B_1 (squares in Fig. 19.1.1), and idler and signal-B_2 (triangles in Fig. 19.1.1), and *triple* coincidences between the idler, signal-B_1 and signal-B_2 (crosses Fig. 19.1.1). If the photon splits at the beam-splitter then we should see triple coincidences. We do not see any. The data is consistent with the photon to interfere with itself and then go to either detector B_1 of detector B_2. The photon goes through the two arms of the interferometer without splitting. Oxymoronic, but that is what we see.

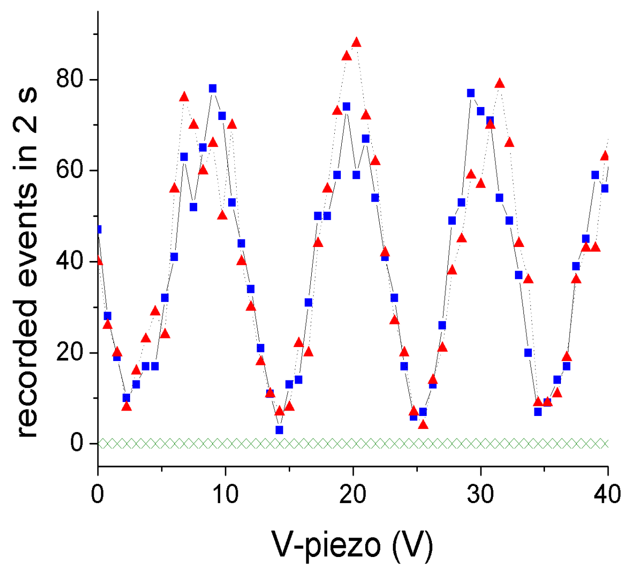

Figure 19.5: Data on the setup of photon interference with two detectors after the interferometer (Fig. 19.4). Squares and triangles are the double coincidences between the idler detector and detectors B_1 and B_2. The crosses are the triple coincidences of all three detectors.

19.1.2 Spookiness of superposition

Before we discuss polarization let's consider one of the striking consequences of superposition in quantum mechanics, in the "bomb" experiment. Consider a Mach-Zehnder interferometer set such that the paths are indistinguishable and such that $\delta = \pi$. Thus the detector at B will detect no photons because $P = 0$ (see Eq. 19.6. If the paths were distinguishable then the probability of detecting a photon at B would be $P = 1/2$. If the paths were distinguishable by means of blocking one of the arms then the probability of detecting a photon at B is $P = 1/4$. In the latter case, 1/2 of the photons will be blocked, 1/4 of the photons will go to port B and 1/4 to port C.

Notice something peculiar about quantum mechanics: probability. There is a probability that the photon will take this path or that path. If we send one photon, we do not really know where it will go. We only know where it is *likely* to go. It is an important distinction between classical and quantum mechanics. Classical mechanics is deterministic, quantum mechanics is probabilistic. But for all its disturbing aspects, quantum mechanics has survived the test of time to become the most successful physical theory ever invented!

Back to the bomb. Suppose that we have an interferometer set for destructive interference (i.e., $\delta = \pi$). Now suppose also that we have a bomb that gets triggered when a single photon hits it. While the paths of the interferometer are indistinguishable no photon reaches B. We now place our trigger-happy bomb in arm 2, as shown in Fig. 19.6. The paths are now distinguishable. We then send one photon. Because of the bomb there is 25% probability that the photon will reach B. The fact that a photon reaches B means that the photon that reaches it "knows" that there is bomb in arm 2 without going through it! It detects the bomb without touching it! Of course, this is not an efficient experiment: there is also 50% chance that it will go kaboom. However, the point is that quantum mechanics has some predictions that sometimes feel like they are impossible, but when we do the experiment the results agree with quantum mechanics.

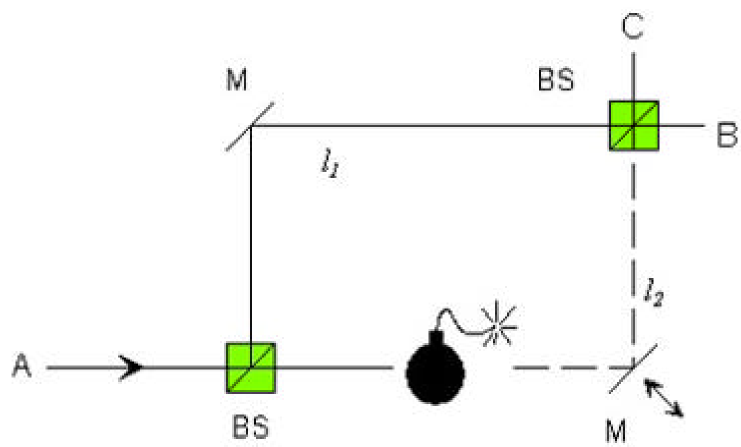

Figure 19.6: Diagram of a Mach-Zehnder interferometer with a bomb in one of its arms.

19.1.3 A to B *vs.* A to C

Earlier we saw that the probability of going from A to B in a Mach Zehnder interferometer (Fig. 19.1) is given by Eq. 19.6:

$$P_{AB} = \frac{1}{2}(1 + \cos\delta). \tag{19.7}$$

But what about the probability for going from A to C? Is there any difference? The answer is yes. A photon going from A to B through arm 1 gets reflected once and transmitted once through the beam splitters. In going through arm 2, the photon gets transmitted once and then reflected once. However, when we go from A to C there is a subtle difference between the two arms. Through arm 1 the photon gets reflected twice, and through arm 2 it gets transmitted twice.

Even if the beam splitters have equal transmission and reflection probabilities there is a difference between the two cases. Let's put it in a different way: there *has* to be a difference. If the probability for going from A to C were also given by Eq. 19.7 then when $\delta = 0$ the probability in going from A to B would be 1, and the probability in going from A to C would also be 1. That is a contradiction! The photon that goes into the interferometer goes to either B or C. The probabilities must be complementary. Therefore, the probability for going from A to either B or C must be 1 always, regardless of how the photon gets there:

$$P = P_{AB} + P_{AC} = 1. \tag{19.8}$$

Combining equations 19.7 and 19.8 we get

$$P_{AC} = \frac{1}{2}(1 - \cos\delta). \tag{19.9}$$

You can also understand this equation in terms of conservation of energy. If there are N input photons then NP_{AB} go to B and NP_{AC} go to C. The photons are going to go here or there, but the total number must remain the same.

Finally, you might ask, why then is the probability for going to B associated with the plus sign and the probability to go to C associated with the minus sign? The difference is a very subtle issue, and depends on the optics (i.e., the beam splitters). It has to do with an added $\pi/2$ phase between the waves that get reflected and transmitted.

Suppose for simplicity that a reflection involves imparting a phase of $\pi/2$ onto the reflected wave. Then the transmitted wave gets no phase upon transmission. In the case when the photon goes from A to B the phase that gets added by reflection gets added equally to both paths so that it cancels out when we take the difference of the two phases. However, in going from A to C the reflection phases add for one path but are not there for the other path. As a consequence the phase difference between the two paths in Eq. 19.5 for A to C is $\delta + \pi$, which gives rise to the minus sign in Eq. 19.9. The important point though is that the probabilities from A to B or C satisfy Eq. 19.8. In a different interferometer the formula for the two cases could reverse.

19.2 Plane waves and polarization

19.2.1 Plane acoustic waves

Ocean waves, from wherever they originate, they expand, and by the time they reach a nice sandy shore they form linear wavefronts. By linear wave fronts we mean that the points that have the same phase form a line, and this line is perpendicular to the direction of propagation of the wave.

Sound waves are not as confined as water waves. Suppose that we are far from a car when it blows its horn. The sound waves leave the car in all directions, but the portion of the sound that reaches us is pretty close to a planar wave front. That is, the regions where the air is compressed form planes, separated by one wavelength, that travel in a direction perpendicular to the plane. This is also known as a "plane wave."

If a plane wave is traveling along the x-axis then its equation is:

$$y = A\cos(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}), \tag{19.10}$$

where A is the amplitude of the wave. In the case of the sound wave A is the pressure at maximum compression of the air molecules. Equation 19.10 has some hidden information: for a given point x at a time t all the points in the plane perpendicular to x (i.e., YZ) have the same phase.

19.2.2 Linearly or plane polarized light

Classically, polarization of light is understood to arise because light is an oscillating electric field and the electric field has a direction that is perpendicular to the direction in which the light wave is traveling.

A possible light wave could be one which travels along the z-direction with an electric field vector oscillating parallel to the y-axis. Such a wave can be represented mathematically by

$$E(z,t) = (0, E_0, 0)\sin(\frac{2\pi}{\lambda}z - \frac{2\pi}{T}t) \tag{19.11}$$

where E_0 is the maximum amplitude of the electric field, and E is the electric field strength at a point z and time t where the wave has a wavelength λ and a period T. Notice that there is no dependence on x or y. Such a wave is also a plane wave like the acoustic wave, only that the amplitude of the wave is given by the electric field of the wave, which happens to be aligned in this case with the y-axis.

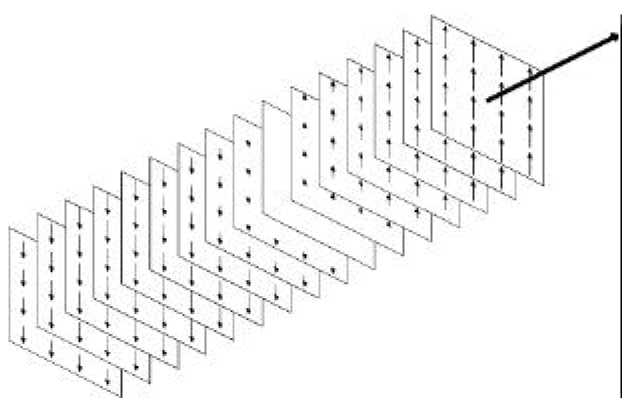

Figure 19.7: A representation of an electromagnetic wave in space. The planes are $\lambda/24$ apart and extend to infinity. The electric field is indicated by the arrows. In a given plane the electric field is the same at every point.

Figure 19.7 represents the electric field of the electromagnetic wave described by Eq. 19.11 as it exists in three dimensions. The small arrows in the figure show the direction and magnitude of the electric field. The planes in the diagram are $\lambda/24$ apart. In a given plane the electric field is the same at every point. Because the direction of the oscillation is perpendicular to the direction of travel shown by the large arrow, this wave like all electromagnetic waves is a transverse wave.

Transverse waves always have the possibility of being polarized. When, as in this example, the electric field always points parallel to the same direction such a wave is said to be "polarized".

Such light is said to be "linearly polarized" because the electric field oscillates in a straight line. Can you imagine a wave linearly polarized in the x-direction? This kind of light is also said to be "plane polarized" because the oscillation takes place in a plane.

Remember that electric field is a vector. The most important concept about vectors is that they can be decomposed into two orthogonal vectors, as shown in Fig. 19.8. The standard trigonometric relations alow us to find the values of the components when we know the magnitude of the vector and the angle that it forms with the orthogonal axes.

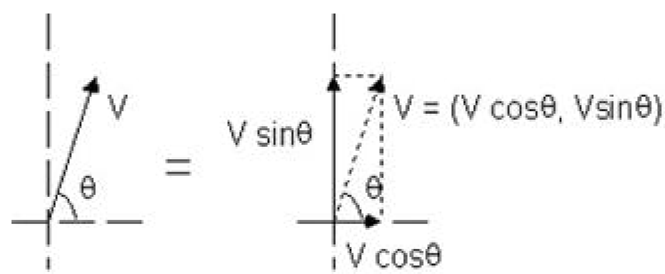

Figure 19.8: A vector V decomposed into components aligned with orthogonal axes.

Exercise 6 Give the component vectors of V_1 and V_2 from Fig. 19.9.

Figure 19.9: Figure for exercise 6.

In our discussion we are going to restrict ourselves to waves that travel in one dimension along an axis. The electric field will then be a vector with some orientation but contained in a plane that is transverse to the propagation direction. If we do experiments in a laboratory where the beams travel in a horizontal plane, then the electric field of the wave is contained in a vertical plane. The simplest orthogonal axes to describe the electric field are then the horizontal axis and the vertical axis. This is shown in Fig. 19.10. Since the electric field in the figure is parallel to the plane of the paper, then we can assume that the light is coming toward us, out of the paper and perpendicular to the plane of the paper.

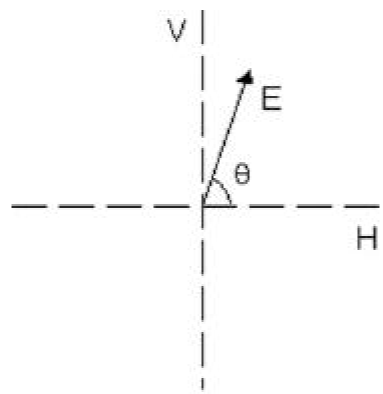

Figure 19.10: Electric field of the light (polarization) in the horizontal-vertical (HV) axes.

Another possibility for reference axes is a set of axes rotated by an angle θ from the HV axes. We will call these the $H'V'$ axes. E could then refer the electric field of the light wave in terms of $H'V'$. Consider the example of Fig. 19.11. The electric field has an amplitude E_0. In the HV axes it is represented as $(0, E_0)$. In the $H'V'$ reference frame it is represented as $(E_0 \sin\theta, E_0 \cos\theta)$.

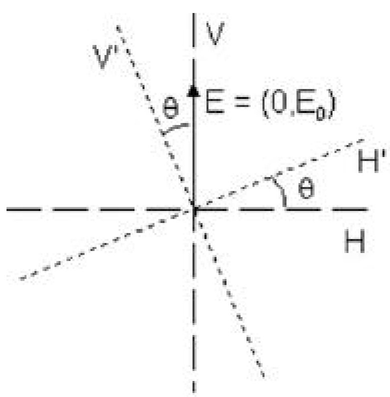

Figure 19.11: Electric field of the light and two axes in which it can be represented.

19.2.3 Polarizers

The light that comes from the Sun or from a lamp is unpolarized. That is, it is made up of waves with polarizations of different orientations. You can produce linearly polarized light by passing unpolarized light through special plastic films called polarizers. Commonly these contain long molecules of polyvinyl alcohol stretched and oriented in the same direction. Iodine atoms are attached to the molecules, and when light is incident on such a sheet, the component of its oscillation parallel to the molecules causes the iodine atoms to move along the molecules, absorbing this component of the incident light. The other component of oscillation is only weakly affected by the sheet and emerges largely unabsorbed. As a result, light emerging from these so-called "polarizing sheets" is linearly polarized perpendicular to the direction of the long molecules.

Polaroid sunglasses are made of this sort of polarizing sheet. Their lenses are oriented to block oscillations parallel to the ground. This is because when light reflects from asphalt on the surface of a road or from water on a lake or pond, the oscillations parallel to the horizon are more efficiently reflected than the others. This is partly what causes glare and setting Polaroid lenses to block this component of the reflected light reduces glare.

A simple experiment is to take two polarizing sheets and put them at right angles. The first absorbs all of one component of oscillating electric field. The second absorbs the rest, and nothing is transmitted. The combination of two sheets at right angles is essentially opaque.

When unpolarized light of intensity I_0 is incident on a polarizer, half of the intensity is transmitted. The other half is absorbed.

19.2.4 Light through polarizers: Wavespeak

Imagine a beam of linearly polarized photons entering a polarization analyzer. As mentioned before, the polarizer absorbs light parallel to an internal axis, called "extinction axis," and transmits light parallel to the orthogonal axis, called the "transmission axis."

Consider the case when the light wave is coming out of the page with the electric field oriented vertically. We also call this "vertically polarized." The light wave is incident on a polarizer. Figure 19.12 shows a diagram representing one case. The polarization of the light is represented by the broad, double-headed arrow as it enters the polarizer. The polarization is represented by the square, with the **solid axis** marked on the analyzer denoting the transmission axis.

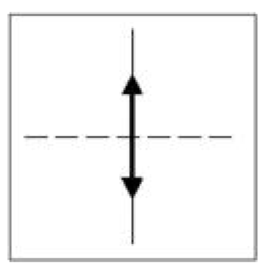

Figure 19.12: Light vertically polarized entering a polarizer with its transmission axis oriented vertically.

Since the polarization of the light is aligned with the transmission axis of the polarizer then all of the light gets transmitted. If the electric field is E_0 then the transmitted electric field is $E_T = E_0$ and its intensity is $I_T = I_0$.

Consider now the case of Fig. 19.13. The incident electric field is perpendicular to the transmission axis. The component of the electric field along that axis is zero. All of the light is aligned along the extinction axis. Therefore, it is all absorbed. The transmitted field and intensity are $E_T = 0$ and $I_T = 0$.

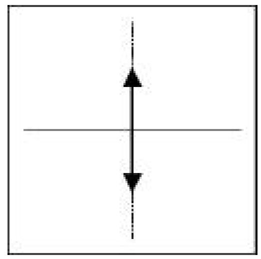

Figure 19.13: Light vertically polarized entering a polarizer with its transmission axis oriented horizontally.

Consider now an intermediate case, shown in Fig. 19.14. The transmission

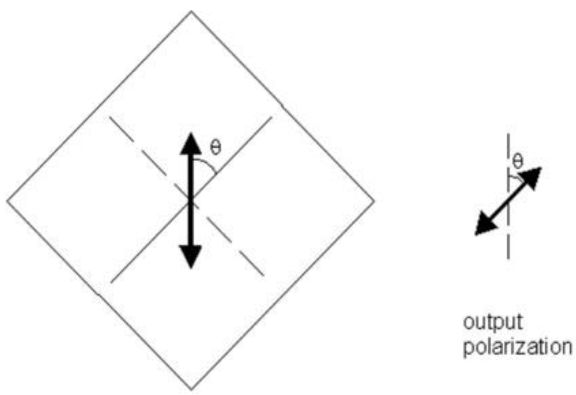

Figure 19.14: Light vertically polarized entering a polarizer with its transmission axis oriented an angle θ with the vertical.

axis is now forming an angle θ with the incident polarization. The incident electric field can be decomposed into two components: one parallel to the transmission axis ($E_0 \cos\theta$), and another parallel to the extinction axis ($E_0 \sin\theta$). The former component gets transmitted and the latter gets absorbed. This is a general case that is worthy to underscore:

1. The transmitted electric field is $E_T = E_0 \cos\theta$.
2. The transmitted intensity is $I_T = I_0 \cos^2\theta$. In optics this is known as Malus' law.
3. The light emerges from the polarizer with a polarization **oriented an angle θ with respect to the incident polarization.**

19.2.5 Light through polarizers: Quantumspeak

For the simple situations described above, the quantum predictions of how much polarization will emerge from a polarization analyzer are the same as the classical predictions. It must be so because experiment shows the classical predictions are correct. However, this must also work at the photon level so we need to see how this gets put into the quantum language.

When you want to know the probability of an outcome, you use quantum mechanics to calculate the probability amplitude and then square it. In connection with interference we have seen that the photon does not split. Rather, the photon goes through the interferometer in a superposition of going through the two arms. We will use that approach here.

Photons are polarized. Suppose that the light is polarized forming an angle θ with the horizontal, as shown in Fig. 19.8. When we talk about waves we can decompose the linearly polarized wave into two waves polarized along the horizontal and vertical axes. We should be able to do something similar at the photon level, but how?

Let's go back to waves: if $\theta = 60°$then the amplitude of the electric field along the horizontal direction is $E_0 \cos 60° = E_0/2$. In the same way, the component along the vertical axis is $E_0 \sin 60° = \sqrt{3}E_0/2$. If we put a polarizer with transmission axis horizontal, the transmitted intensity will be $I_0/4$ (i.e., the square of the field). If we rotate the polarizer to be vertical we will obtain a transmitted intensity $3I_0/4$. If we think in terms of photons, then we should detect 1/4 of the incident photons when the polarizer is horizontal and 3/4 when the polarizer is vertical. Following the relation between intensity and probability that we used in the interference section we can say that the probability of getting a photon transmitted is 1/4 when the transmission axis of the polarizer is aligned with the horizontal, and 3/4 when it is aligned with the vertical. This has all the ingredients of superposition! We can then say that *a photon with polarization oriented at an angle can be represented as being in a superposition of being polarized horizontal and vertical, with amplitudes given by the components of the polarization along those axes* ($\cos\theta$ *for horizontal and* $\sin\theta$ *for vertical).*

If we now send a photon to a polarizer with its orientation forming an angle θ with the transmission axis, then quantum mechanically we can state that it is in a superposition of two polarizations. One polarization is parallel to the transmission axis and another perpendicular to the transmission axis. As a result we can state:

1. The absolute value of the amplitude for the photon to be transmitted is $\cos\theta$.

2. The probability that the photon is transmitted is $\cos^2\theta$.

3. The polarization of the transmitted photon has *a new orientation*: the orientation of the transmission axis of the polarizer.

The last conclusion above is very striking. The transmission of the photon through the polarizer involves *projecting it* into a new state with a new polarization.

19.2.6 A new quantum mystery

Just when you thought it was starting to clear up, here is a new mystery. Consider a photon polarized vertically. If we put a polarizer with a transmission axis oriented horizontally no photons go through. Let's now put before the horizontal polarizer another one oriented an angle θ with respect to the vertical. Are photons transmitted past the two polarizers?

The answer is yes! Check this out: the first polarizer at an angle theta with the incident polarization projects the polarization of the transmitted photon. this new orientation forms an angle $(\pi/2 - \theta)$ with the horizontal polarizer that follows, so some photons get transmitted. The amplitude of passing through the first polarizer is $\cos\theta$. The amplitude of passing through the second (horizontal) polarizer is $\cos(\pi/2 - \theta)$. This situation is shown in Fig. 19.15.

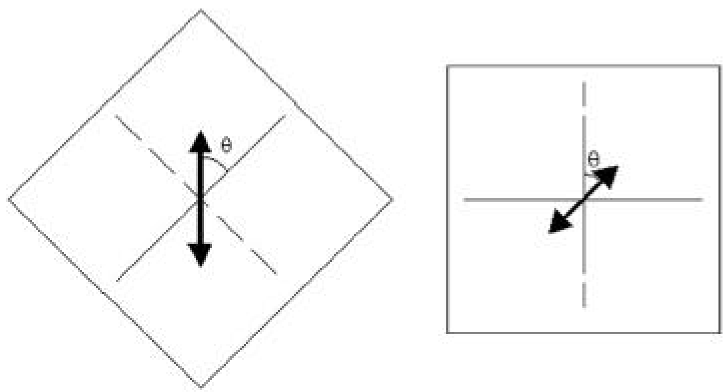

Figure 19.15: Light vertically polarized entering a polarizer with its transmission axis oriented an angle θ (left). The transmitted beam then is incident on a polarizer with its transmission axis oriented horizontally.

Exercise 7 A vertically polarized photon is incident on a pair of polarizers The first one has its transmission axis oriented an angle θ with the vertical. The second one's transmission axis is oriented horizontally.

1. What is the probability that a photon be transmitted through the two polarizers when $\theta = 30°$? (Ans: 3/16)

2. What is the final polarization orientation of the photon? (Ans: horizontal)

19.3 The Quantum Eraser

Before we discuss an new puzzle, lets consider two photons that are linearly polarized, have the same wavelength and travel in the same direction. If their polarizations are parallel to each other, we cannot tell them apart: they are indistinguishable. However, if their polarizations are perpendicular to each other then we can tell them apart. Suppose that one is polarized vertically and the other horizontally. Putting a polarizer with its transmission axis vertical then transmits the vertically polarized one but not the horizontally polarized one. Conversely, polarizer oriented horizontally will transmit the horizontal one but not the vertical one. Thus, photons in orthogonal polarizations can be distinguish by means of a polarizer. We can say that these two photons contain distinguishing information encoded in their polarization.

Consider the interferometer shown in Fig. 19.16. We will consider an experiment in three steps. The light entering the interferometer is polarized vertical

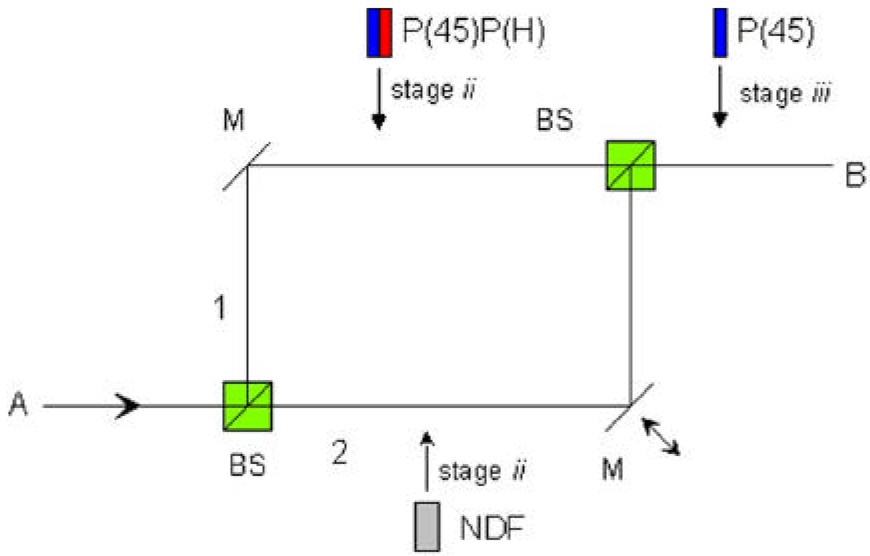

Figure 19.16: Experimental layout for the experiment on the "quantum eraser."

(i.e., perpendicular to the page). In stage (i) the interferometer is aligned to have indistinguishable paths for the photon to go from A to B. Thus we get interference.

In stage (ii) we place two polarizers in arm 1. One polarizer with transmission axis forming 45°with the vertical and a second one with transmission axis horizontal.

Exercise 8 What is the transmission amplitude to go from A to B through arm 1? (You will first have to calculate the probability amplitude of going through the two polarizers.) (Ans: 1/4)

We now put a neutral density filter that provides the same attenuation as the two polarizers (i.e., with a transmission probability amplitude of 1/2). When we do the experiment we find no interference. This is because the two arms are now distinguishable by means of the polarization of the light.

Exercise 9 Find an expression for the probability for going from A to B (both arms) in this case. (Ans: 1/8)

In stage (iii) we put a polarizer after the interferometer with its transmission axis forming 45°with the vertical axis.

Exercise 10 For stage (iii) of the experiment

1. Find the absolute value of the probability amplitude for going from A to B through arm 1. (Ans: $\sqrt{2}/8$)

2. What is the orientation of the polarization reaching B coming from arm 1? (Ans: +45°relative to the horizontal)

3. Find the absolute value of the probability amplitude for going from A to B through arm 2. (Ans: $\sqrt{2}/8$)

4. What is the orientation of the polarization reaching B coming from arm 2? (Ans: $+45°$ relative to the horizontal)

If you answer the above questions, you will conclude that the polarizer placed after the interferometer *erases* the distinguishing information. Interference should be regained! We will investigate this in the laboratory.

Exercise 11 Find an expression for the probability for going from A to B in stage (*iii*) as a function of δ. (Ans: $(1/16)(1+\cos\delta)$)

19.4 Problems

Problem 1 When a laser pointer is projected on a screen we see a red spot ($\lambda = 670$ nm). The intensity of the light reaching the screen is 3 mW.

1. Find the number of laser photons reaching the screen per unit time.

2. If we think of these photons as a long stream of short bursts of light that are evenly separated. How far apart are they?

Problem 2 A weak source of light of wavelength 512 nm illuminates a pair of slits. The source sends an average of 1000 photons every second.

1. Find the intensity of this light source in mW.

2. If we think of these photons as a long stream of short bursts of light that are evenly separated. How far apart are they? (Hint: How fast are they going?)

Problem 3 Consider the Mach-Zehnder interferometer shown in Fig. 19.1. Each beam-splitter reflects half and transmits half of the intensity of the light that is incident on it. The interferometer has two output ports B and C.

1. The intensity of the incident light is I_0. If we block one of the arms, what is the intensity of the light exiting the interferometer ports (in terms of I_0)?

2. Both arms are now unblocked. The intensity of the light exiting through port B is $I_B = (I_0/2)[1+\cos\delta]$, where δ is the phase difference between the beams from the two arms due to the difference in length of the two arms. Find an expression for the intensity exiting through port C in terms of I_0 and . Hint: energy must be conserved.

3. The wavelength of the light is 900 nm. Find when the difference in length of the two arms (i.e., $l_1 - l_2$) is 2 m.

4. Consider a photon going from A into the interferometer. The lengths of the two arms are now the same (i.e., $l_1 = l_2$).

   (a) If the paths are indistinguishable, what is the probability that the photon will be detected at B?

(b) If the paths are indistinguishable, what is the probability that the photon will be detected at C?

(c) If the paths are distinguishable, what is the probability that the photon will be detected at B?

(d) If the paths are distinguishable, what is the probability that the photon will be detected at C?

Problem 4 Consider the Mach-Zehnder interferometer shown in Fig. 19.1. It has 50-50 beam splitters, which reflect 50% of the intensity of the light incident on them, and transmit the other 50%.

1. Find the probability of transmission through a 50-50 beam splitter.

2. Find the he absolute value of the probability amplitude of going through the beam splitter.

3. The probability amplitude of going through the two beam splitters is the product of the probability amplitudes. What is the absolute value of the probability amplitude for going from A to B when arm 1 is blocked?

4. What is the probability of a photon to go from A to C when arm 2 is blocked?

5. If the two paths of the interferometer are **distinguishable**,

   (a) What is the probability of going from A to B when neither arm is blocked?

   (b) What is the probability of going from A to C when neither arm is blocked?

6. If the paths through the interferometer are **indistinguishable**,

   (a) What is the probability for a photon to go from A to B when $\delta = 3\pi$?

   (b) If 500 photons per second are incident to the interferometer from A. How many photons reach B in one second when $\delta = 3\pi/2$?

   (c) If 500 photons per second are incident to the interferometer from A. How many photons reach B when $\delta = 3\pi$?

   (d) Based on the previous quesiton, how many photons reach C in one second when $\delta = 3\pi$?

   (e) If 500 photons per second are incident to the interferometer from A. What is the value of δ if 250 photons reach B in one second?

Problem 5 Consider the interferometer in Fig. 19.17, It has a neutral density filter with a transmission **amplitude** of 1/2 in arm 1.

1. What is the absolute value of the probability amplitude of going from A to B via arm 1.

2. What is the probability of going from A to B when $\delta = 4\pi$?

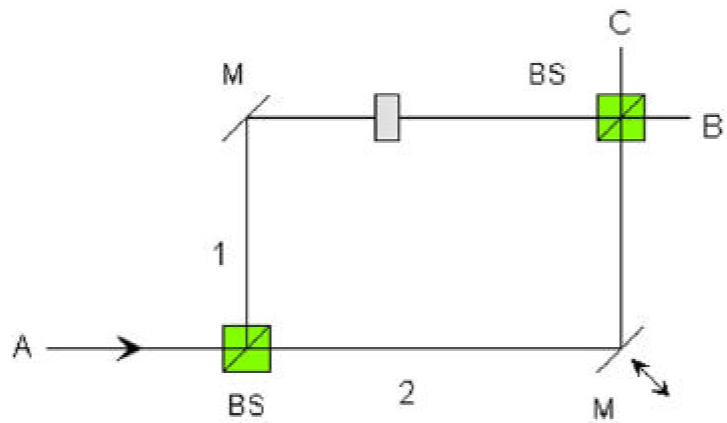

Figure 19.17: Diagram of a Mach-Zehnder interferometer with a neutral density filter in arm 1.

Problem 6 A vertically polarized photon encounters a polarizer forming an angle θ with the vertical axis, followed by a horizontal polarizer. Show that when $\theta = \pi/4$ the probability of transmission through the two polarizers is a maximum. What is that probability?

Problem 7 Use a polarizer to polarize the light from a light bulb. What fraction of the intensity will pass through a second polarizer placed at 30 to the direction of polarization?

Problem 8 A vertically polarized photon is incident onto a polarizer with a transmission axis oriented an angle of 80 degrees counter-clockwise from the horizontal.

1. What is the probability amplitude of the photon to go through the polarizer.

2. We now put a second polarizer that has a transmission axis oriented 10 degrees clockwise from the transmission axis of the previous polarizer.

   (a) What is the probability that an incident photon is transmitted through the two polarizers?

   (b) If we add a third polarizer. Find the orientation of its transmission axis so that no photon is transmitted.

Problem 9 In doing a new quantum eraser experiment we rotate the polarization of the light going through one of the arms of the Mach-Zehnder interferometer using three polarizers. The photons that go into the interferometer are vertically polarized. See Fig 19.18

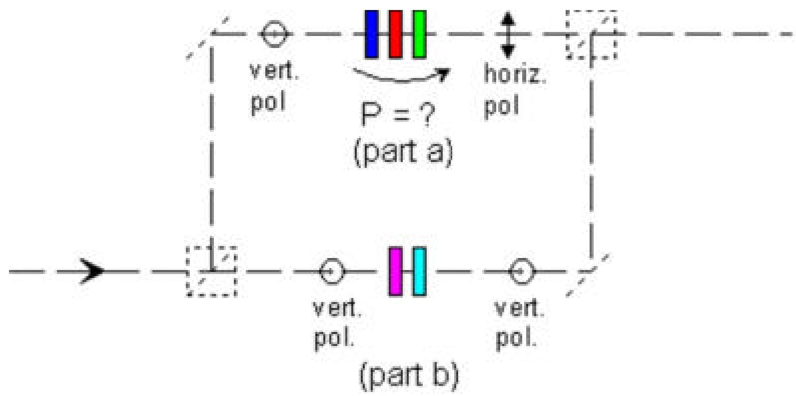

Figure 19.18: Diagram of a Mach-Zehnder interferometer with polarizers for Problem .

1. In the polarization-rotating arm the transmission axis (TA) of the first polarizer is oriented 30 degrees with the vertical, the TA of the second polarizer is oriented 60 degrees relative to the vertical, and the TA of the third polarizer is oriented horizontally. What is the probability that the photon will go through all three polarizers?

2. In the other arm of the interferometer we want to keep the orientation of the polarization vertical but we want to provide a probability amplitude for transmission through the two polarizers that is the same as the one in the other arm. Explain how we can do this with two polarizers, indicating the angle that each polarizer makes with the vertical. Make a diagram.

Problem 10 A Mach Zehnder interferometer has beam-splitters that have an uneven ratio of reflection to transmission. The reflection probability is 1/3 and the transmission probability is 2/3. The 900-nm incident light is vertically polarized (see Fig. 19.18).

1. Find the probability amplitude for a photon to go from A to B via arm 1.

2. We now put a "half wave plate" (HWP) in arm 1 so that it rotates the polarization by 90 degrees. Arm 2 has compensating plate that does not rotate the polarization. Both components transmit all of the light that reaches them. Find the probability that the photon reaches B (both arms).

3. We now put a polarizer after the interferometer oriented at 45 degrees with the horizontal. Find the probability for the photon going from A to B when the lengths of the two arms are the same.

4. If we now increase the length of one of the arms by 1350 nm. What is the probability of going from A to B?

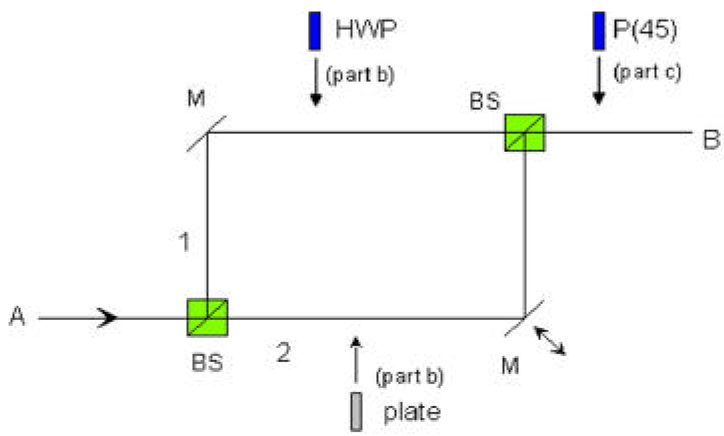

Figure 19.19: Diagram of a Mach-Zehnder interferometer for Problem .